

Toward an Electromagnetic Information Theory for Reconfigurable Microwave Systems

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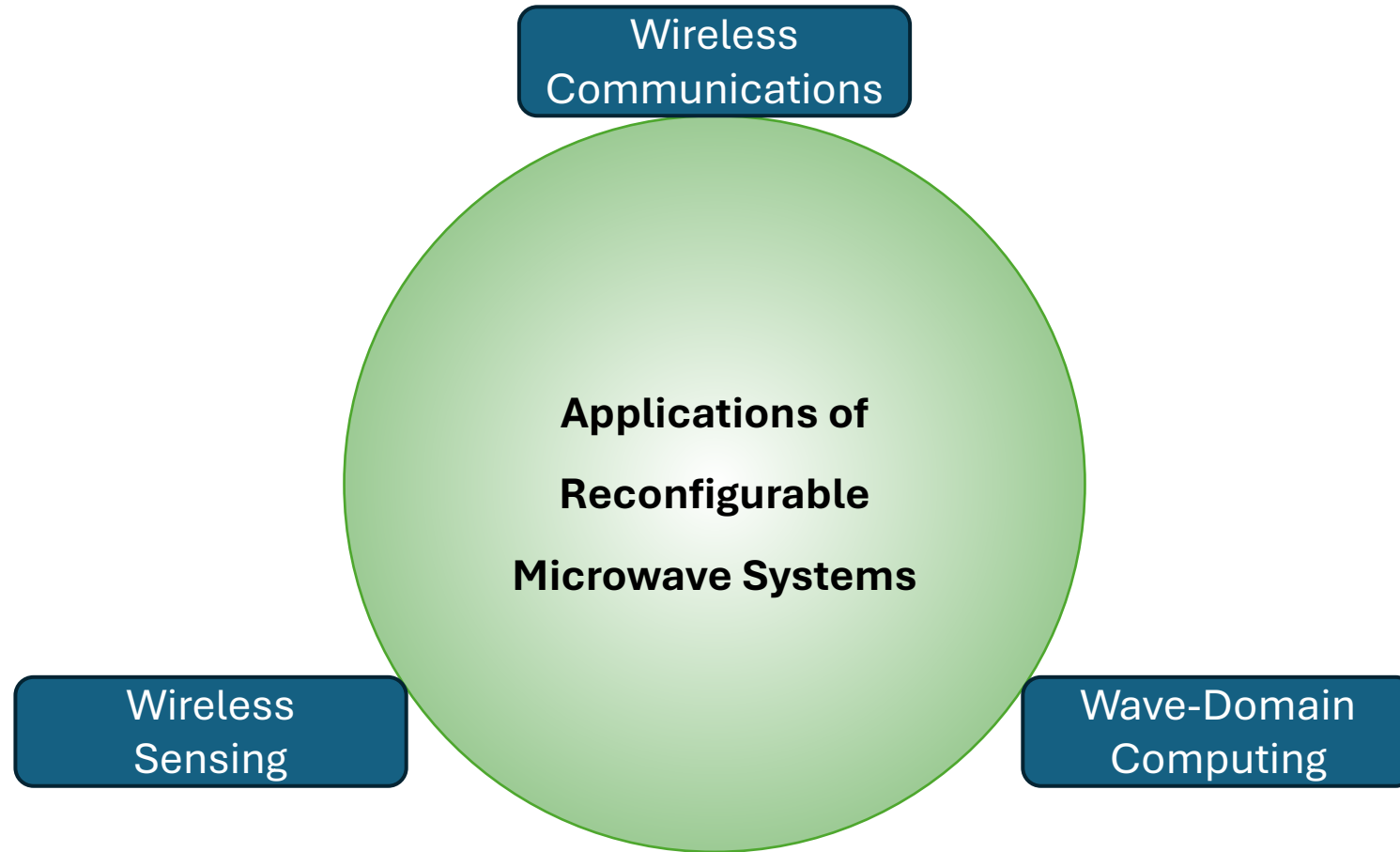
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- 1) **Multiport-network model formulation** for reconfigurable microwave systems
- 2) **Prototype-aware model calibration** for reconfigurable microwave systems
- 3) **Toward a prototype-aware EM information theory** for reconfigurable microwave systems
 - Bounds on multiplexing gain in backscatter MIMO systems
 - Bounds on information transfer via RIS-parametrized SISO channel
 - Bounds on MIMO operator synthesis

Context



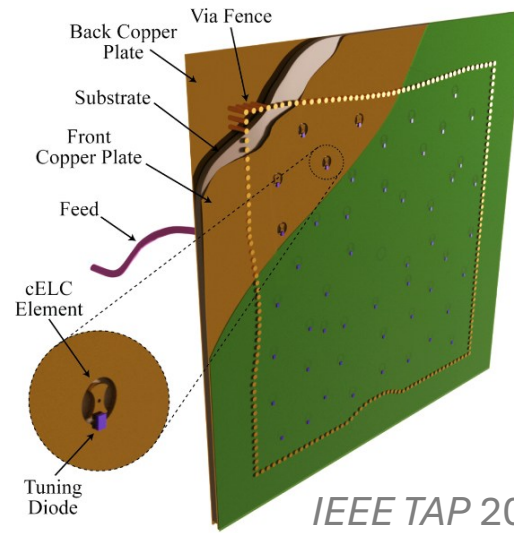
Examples of Extremely Reconfigurable Microwave Systems

Smart Radio Environment



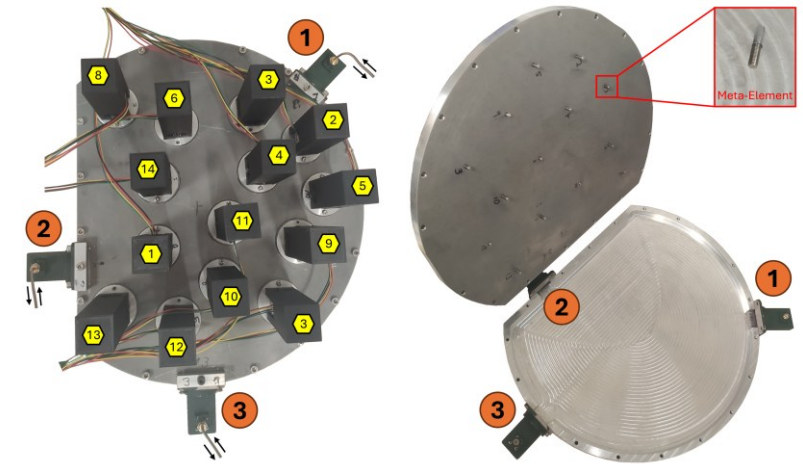
Nat. Commun. 2020

Dynamic Metasurface Antenna



IEEE TAP 2020

Reconfigurable Signal Router




Adv. Sci. 2025

Common features:

- Linear system transfer function: $\mathbf{y} = \mathbf{H}\mathbf{x}$
- \mathbf{H} is parametrized by the system configuration \mathbf{c}

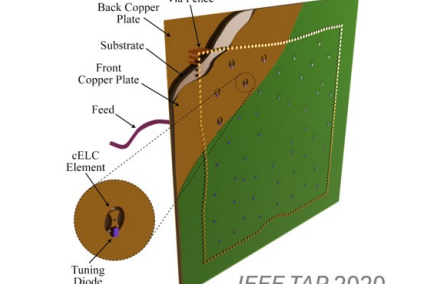
Overarching Questions

Smart Radio Environment



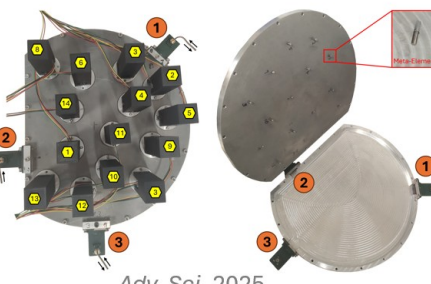
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Adv. Sci. 2025

Common features:

- Linear-system transfer function: $\mathbf{y} = \mathbf{H}\mathbf{x}$
- \mathbf{H} is parametrized by the system configuration \mathbf{c}

Forward Problem:

What is \mathbf{H} for a given \mathbf{c} ?

Inverse Design Problem:

What \mathbf{c} closely approximates a desired \mathbf{H} ?

→ How to formulate a model for $\mathbf{H}(\mathbf{c})$?

→ How to calibrate the model for $\mathbf{H}(\mathbf{c})$?

Electromagnetic Information Theory:

What are the (EM-consistent) limits on communications, sensing and computing via deterministically programmable channels?

Hardware Design Principles:

How to maximize the wave-domain flexibility?
(i.e., the ability of \mathbf{c} to shape \mathbf{H})

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1) Multiport-network model formulation for reconfigurable microwave systems

At a high abstraction level, most reconfigurable microwave systems are amenable to a **universal** multiport network representation because their reconfigurability relies on tunable lumped elements.

Approach: - Partition system into 3 entities:

1) N_A antenna ports (injection & reception of waves)

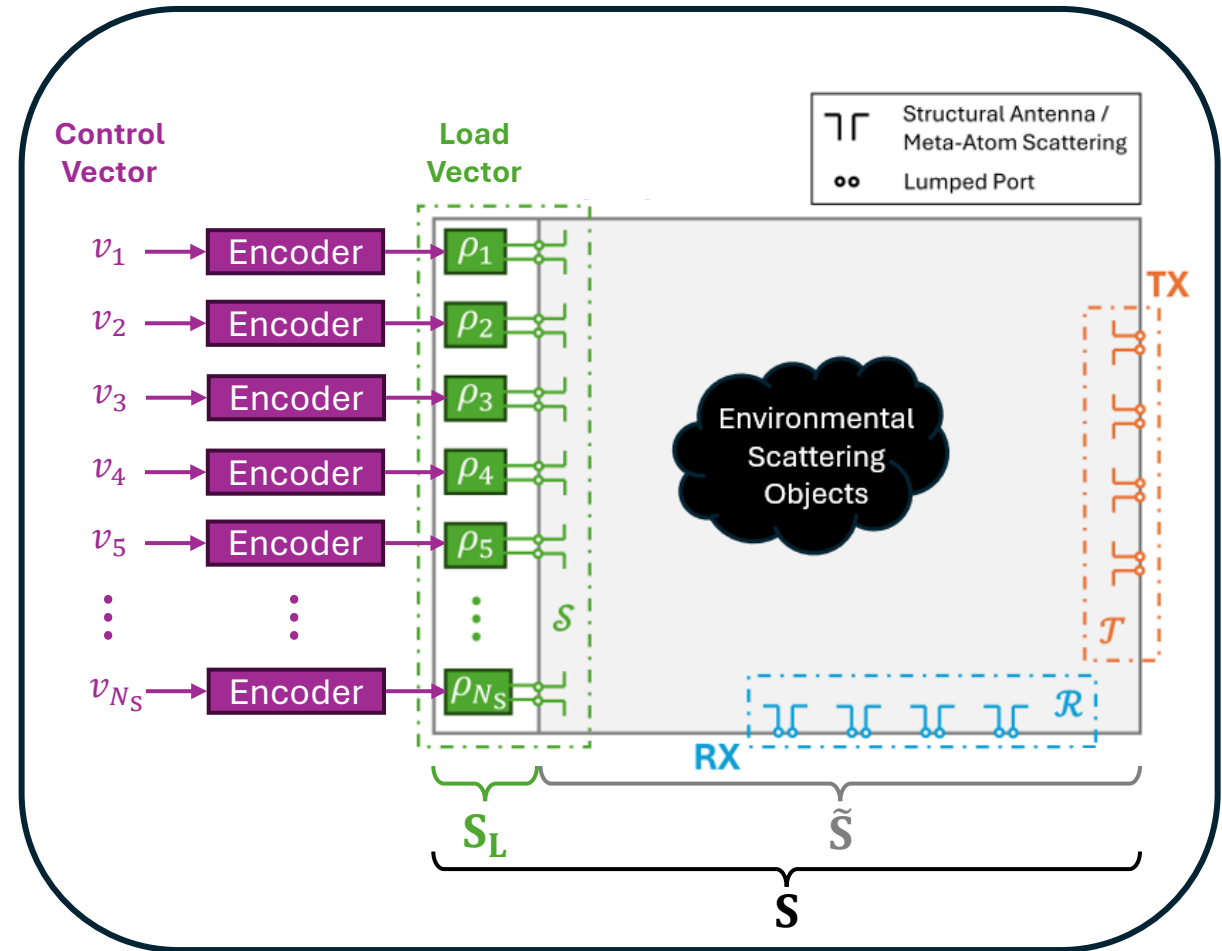
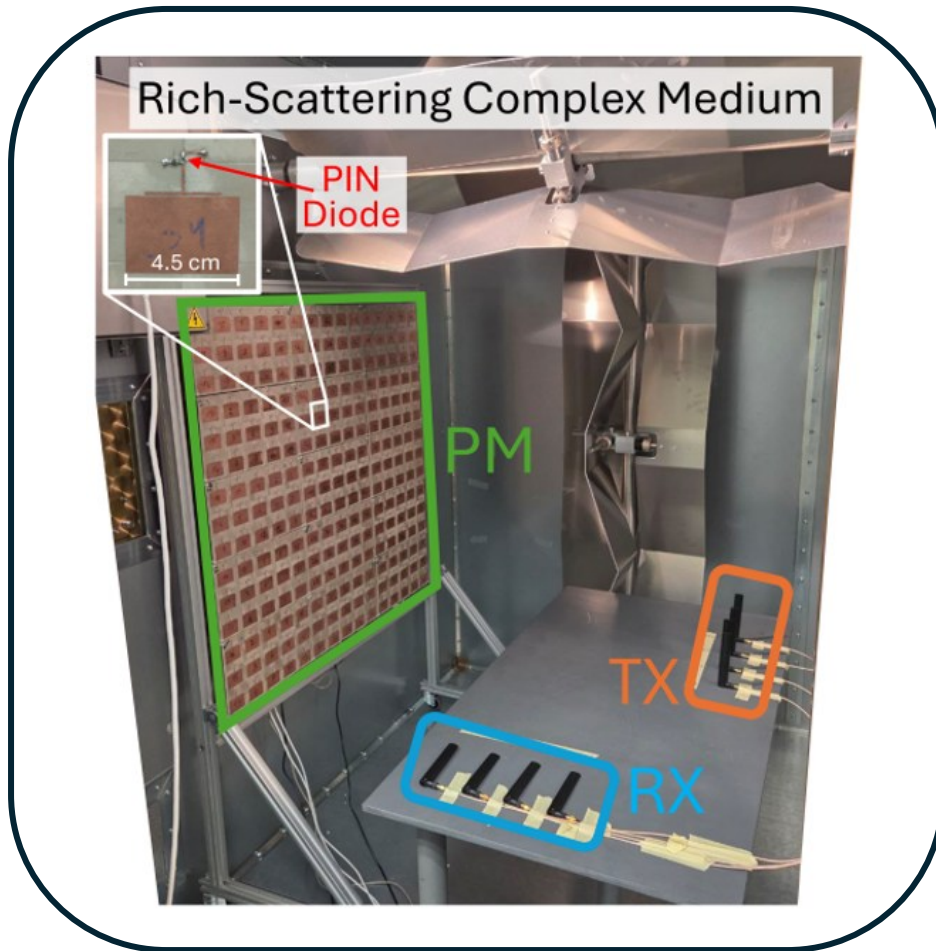
2) N_S tunable lumped elements (e.g., PIN diodes)

3) Static scattering objects (metallic or dielectric, including all “structural scattering” and all environmental scattering)

- A tunable lumped element can be viewed as a “virtual” port terminated by tunable load.

Note: This approach can be applied to **any wave system parametrized by tunable lumped elements**, including RISs, DMAs, BD-RISs, BD-DMAs, SIMs, FIMs, etc.

1) Multiport-network model formulation for reconfigurable microwave systems



1) Multiport-network model formulation for reconfigurable microwave systems

$$\mathbf{S}_L = \text{diag}(f(v_1), f(v_2), \dots, f(v_{N_S}))$$

$$= \text{diag}(\rho_1, \rho_2, \dots, \rho_{N_S})$$

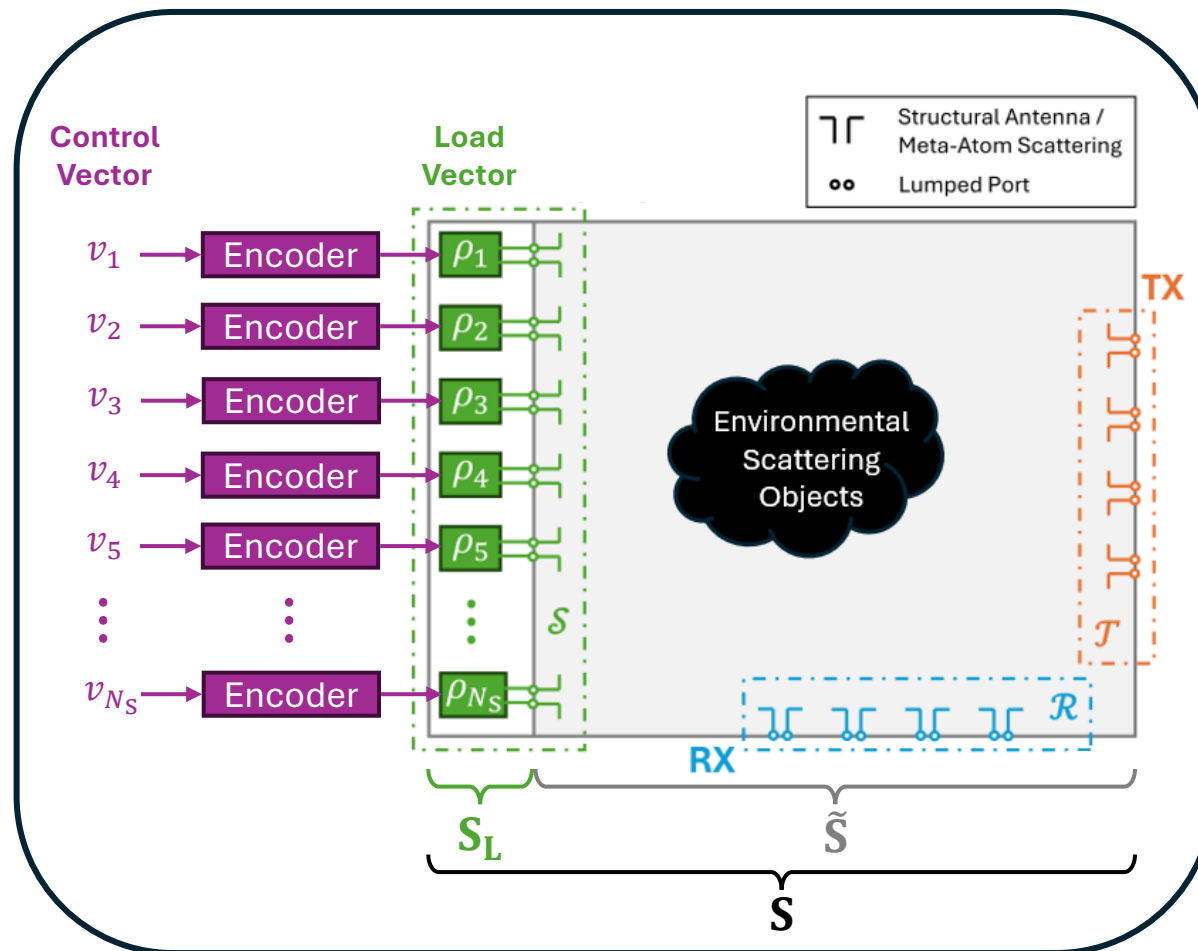
$$\mathbf{S} = \tilde{\mathbf{S}}_{AA} + \tilde{\mathbf{S}}_{AS} (\mathbf{S}_L^{-1} - \tilde{\mathbf{S}}_{SS})^{-1} \tilde{\mathbf{S}}_{SA}$$

$$= \tilde{\mathbf{S}}_{AA} + \tilde{\mathbf{S}}_{AS} \left(\sum_{k=0}^{\infty} (\mathbf{S}_L \tilde{\mathbf{S}}_{SS})^k \right) \mathbf{S}_L \tilde{\mathbf{S}}_{SA}$$

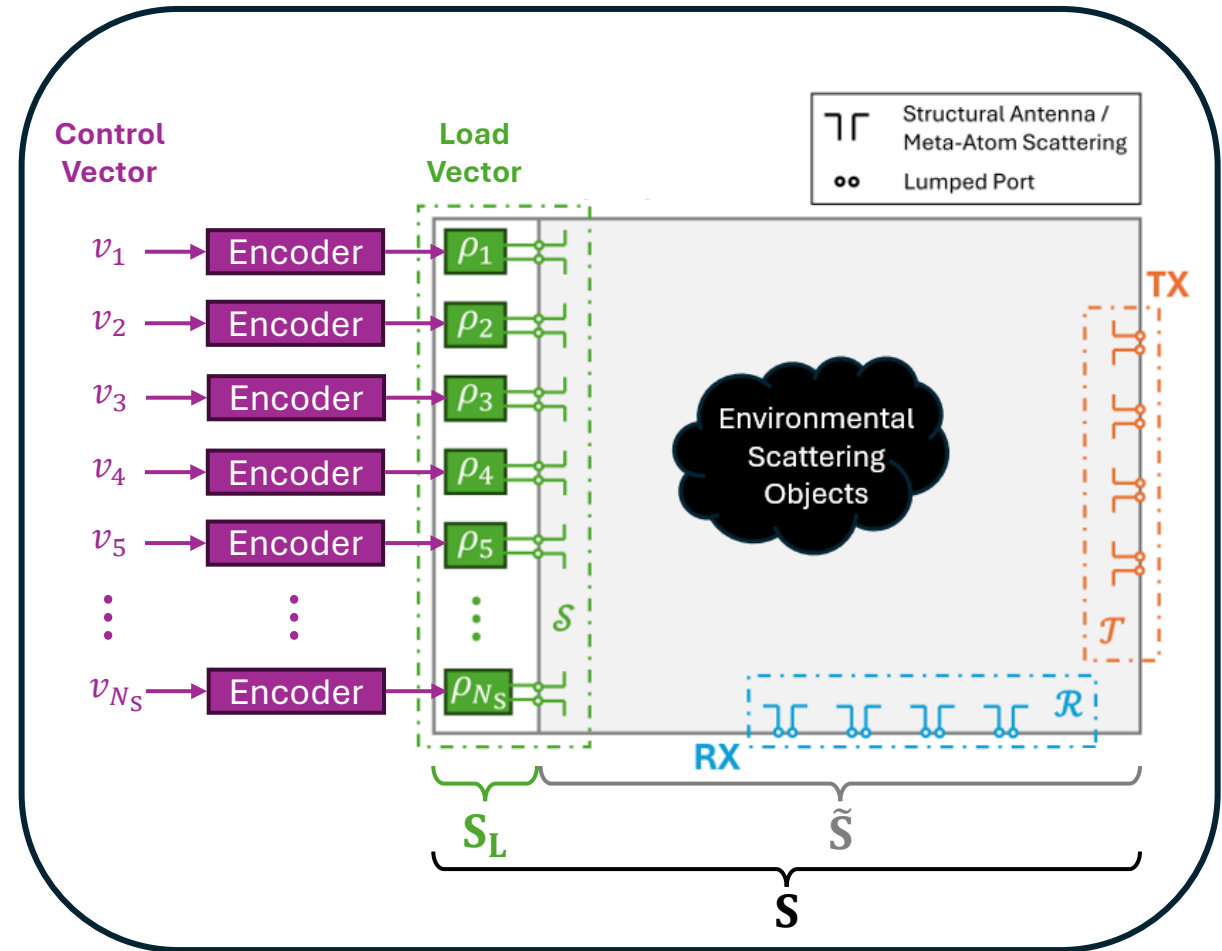
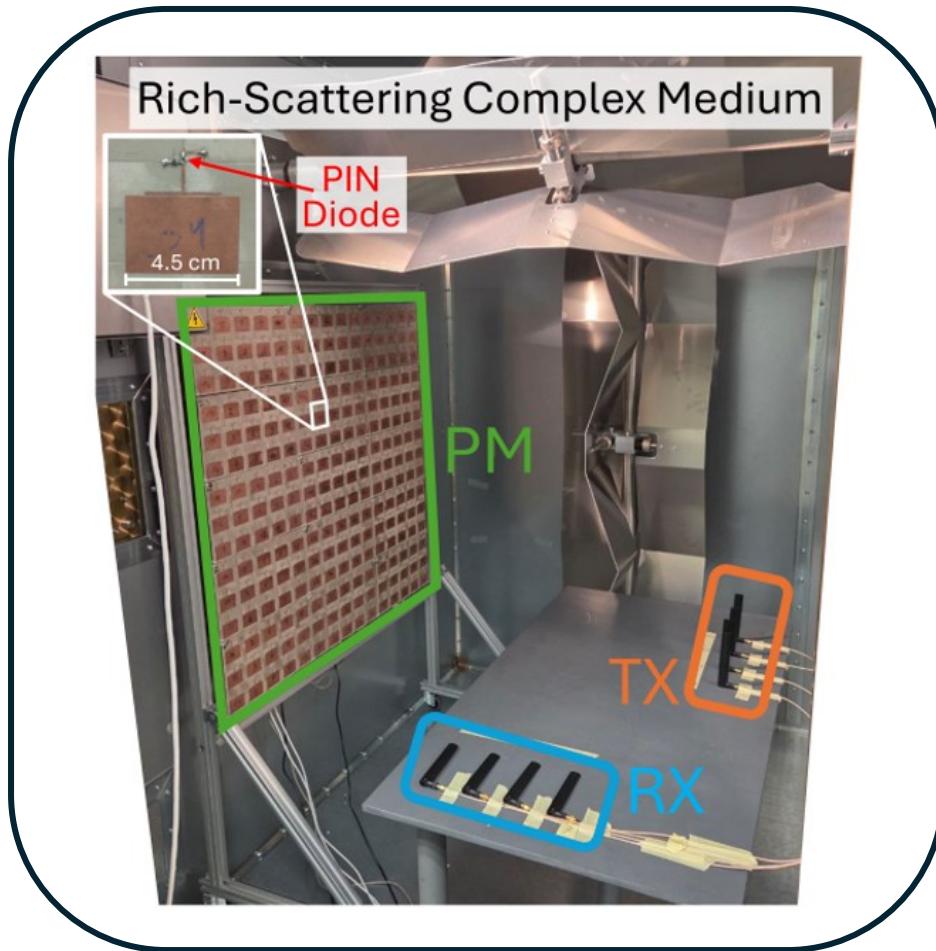
$$\mathbf{H} = \mathbf{S}_{RT}$$

$$= \tilde{\mathbf{S}}_{RT} + \tilde{\mathbf{S}}_{RS} (\mathbf{S}_L^{-1} - \tilde{\mathbf{S}}_{SS})^{-1} \tilde{\mathbf{S}}_{ST}$$

\mathcal{T}	Set of transmitting antenna port indices
\mathcal{R}	Set of receiving antenna port indices
$\mathcal{A} = \mathcal{T} \cup \mathcal{R}$	Set of antenna port indices
\mathcal{S}	Set of meta-atom "virtual" port indices
f	Encoder function

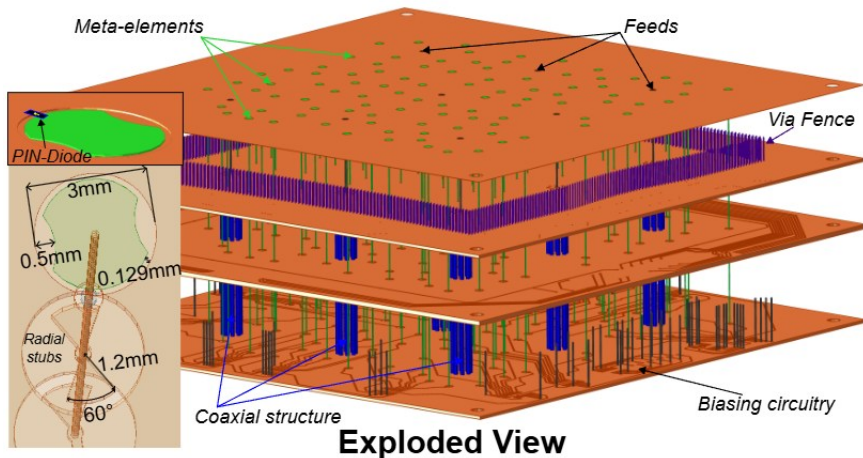
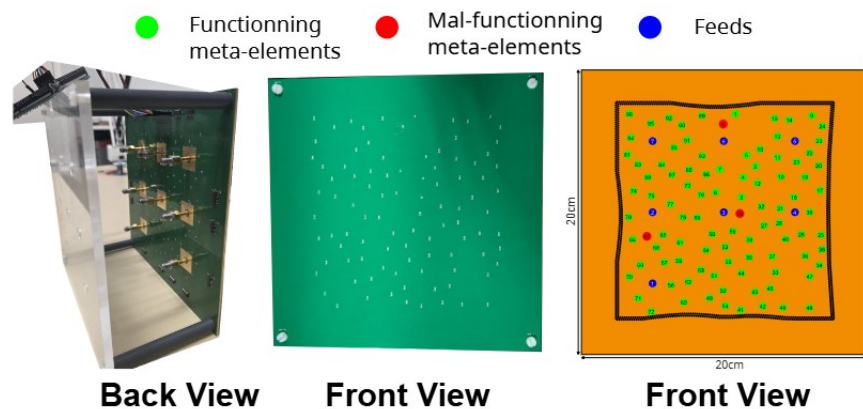


1) Multiport-network model formulation for reconfigurable microwave systems

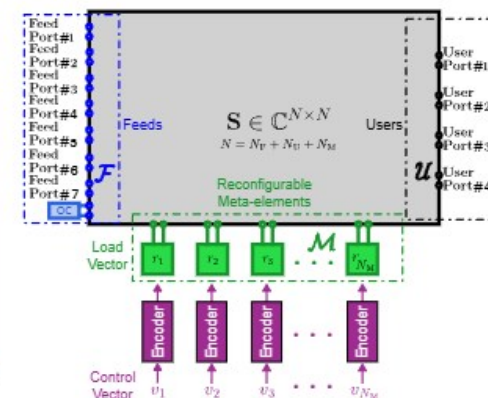
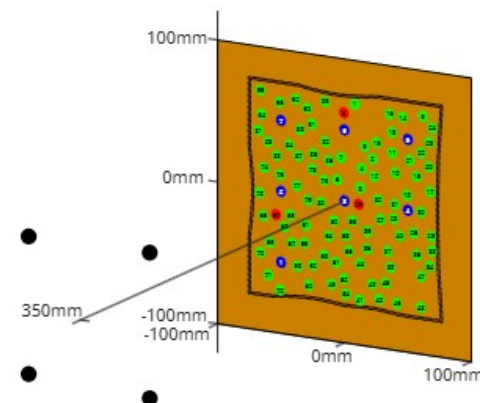
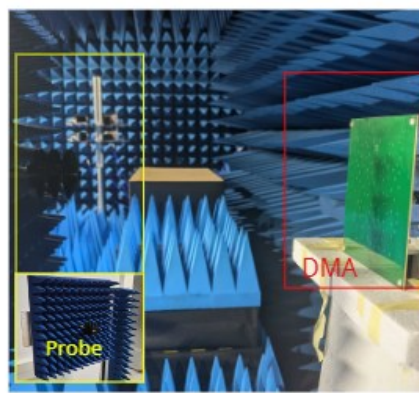


1) Multiport-network model formulation for reconfigurable microwave systems

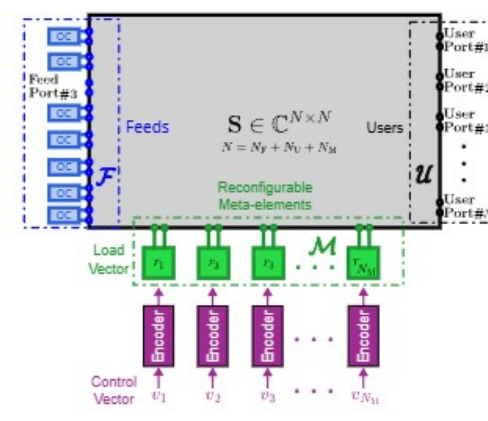
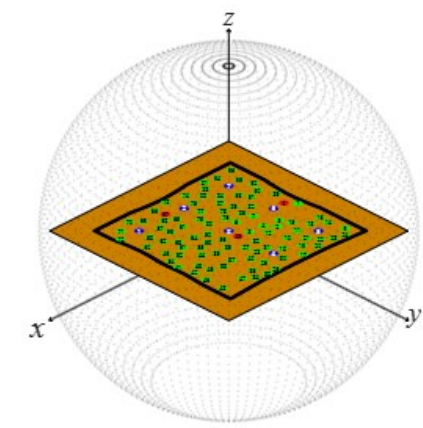
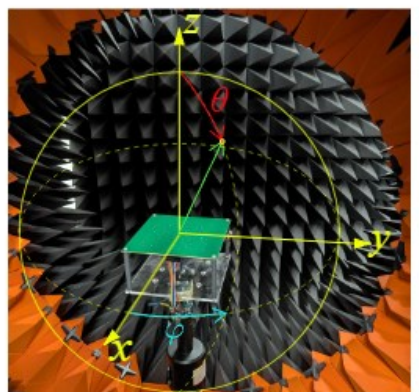
MNT model for a chaotic-cavity-backed dynamic metasurface antenna



Setup 1



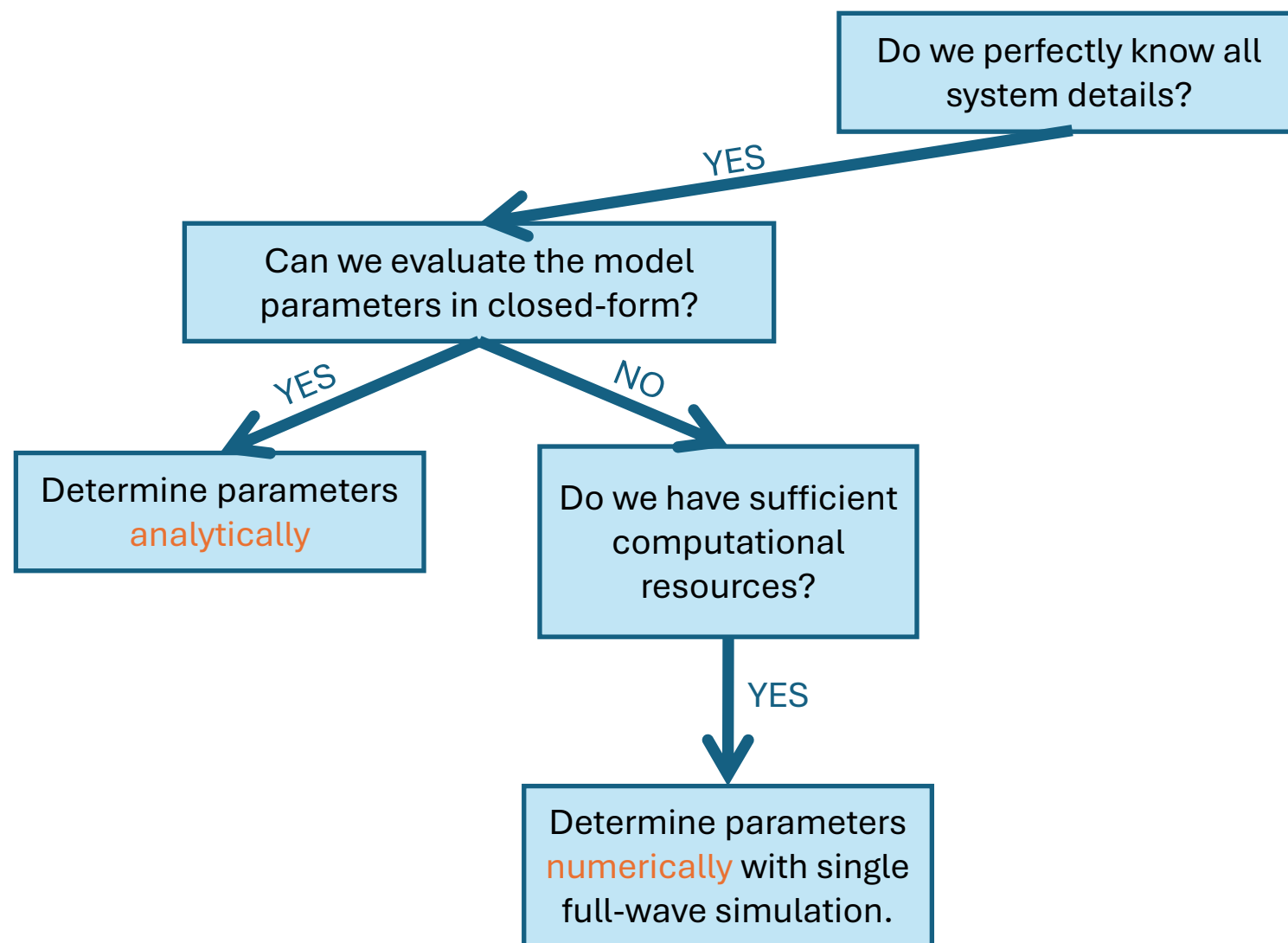
Setup 2



Outline

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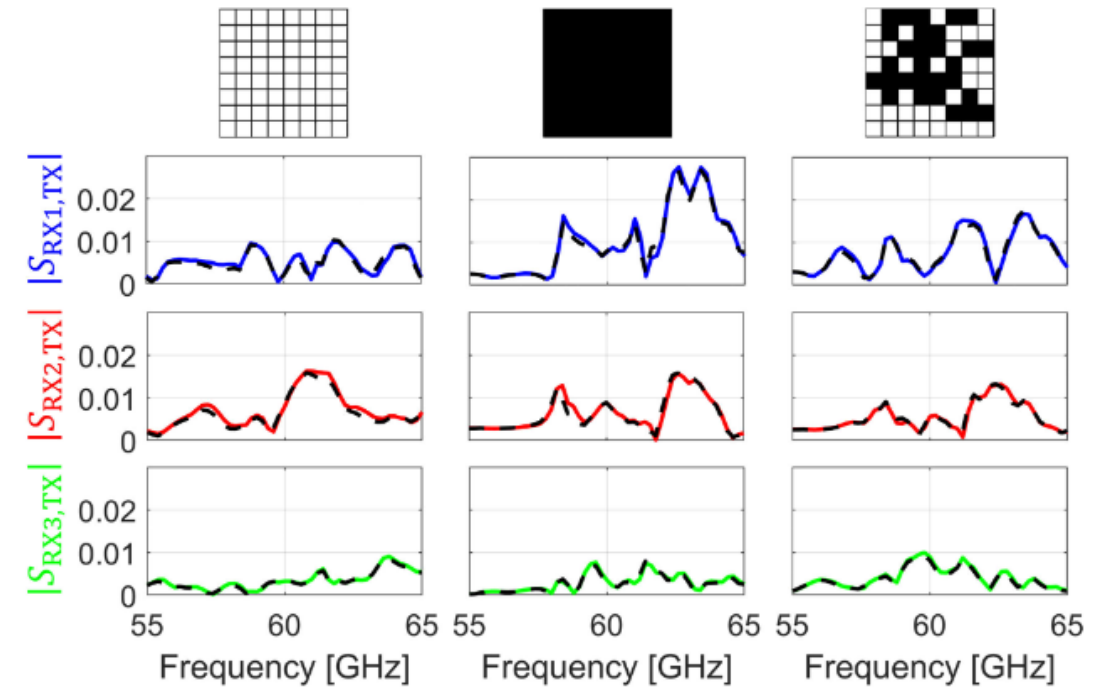
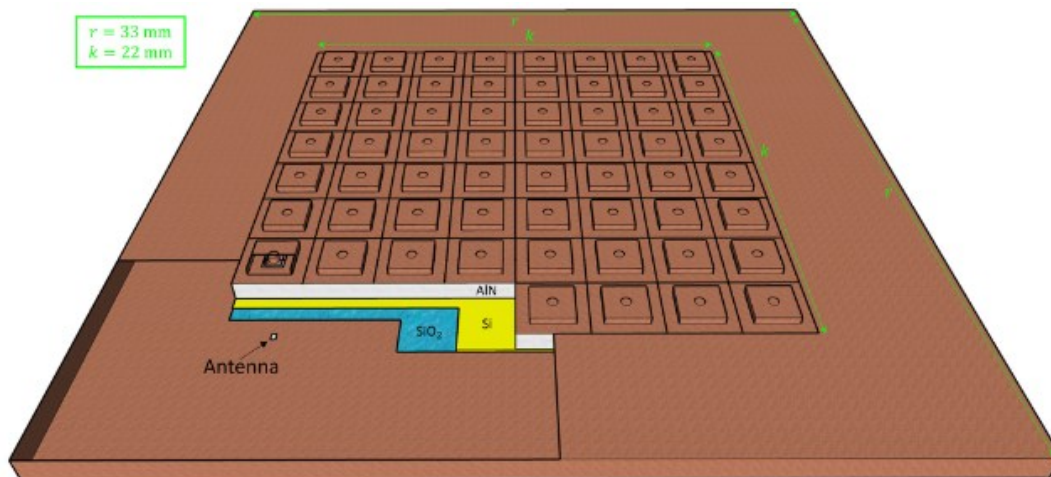
2) Prototype-aware model calibration for reconfigurable microwave systems



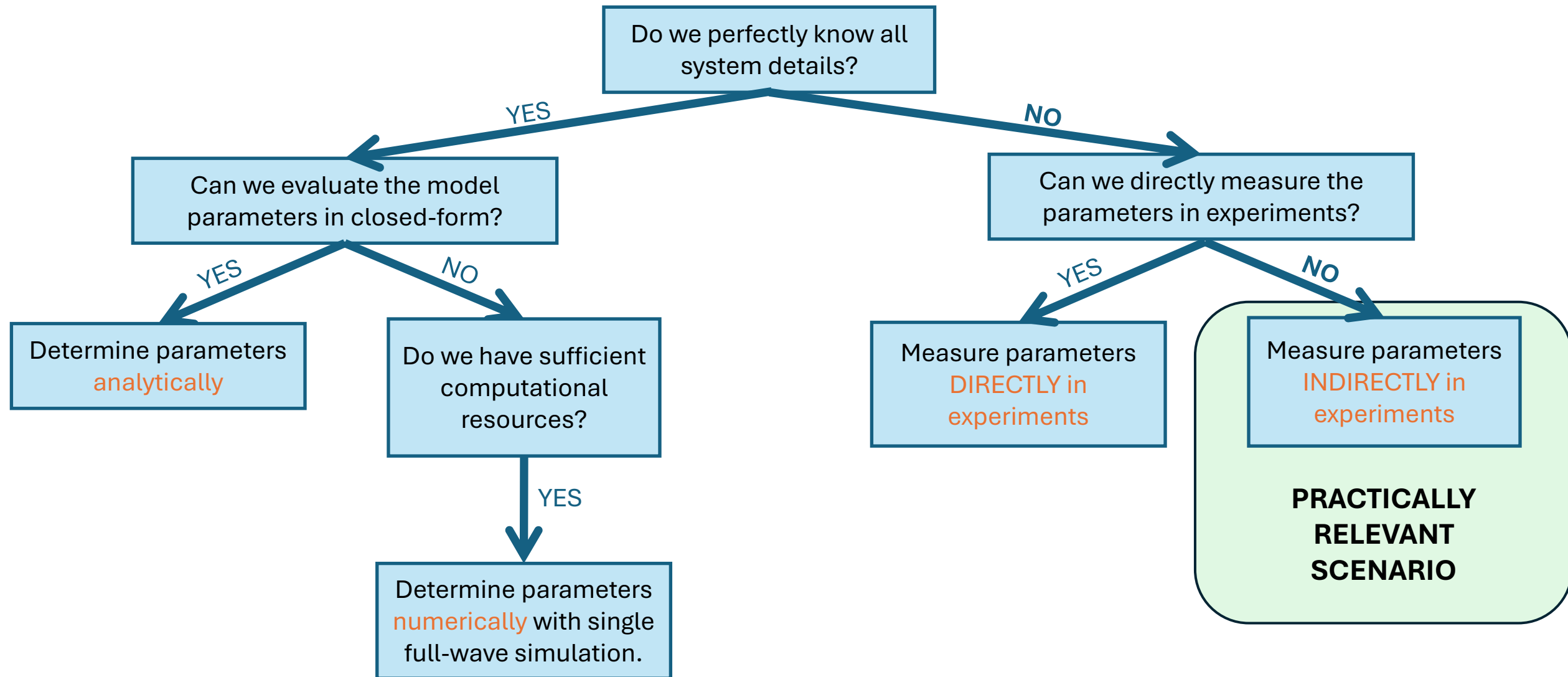
2) Prototype-aware model calibration for reconfigurable microwave systems

Example: RIS-parametrized Wireless Network-on-Chip

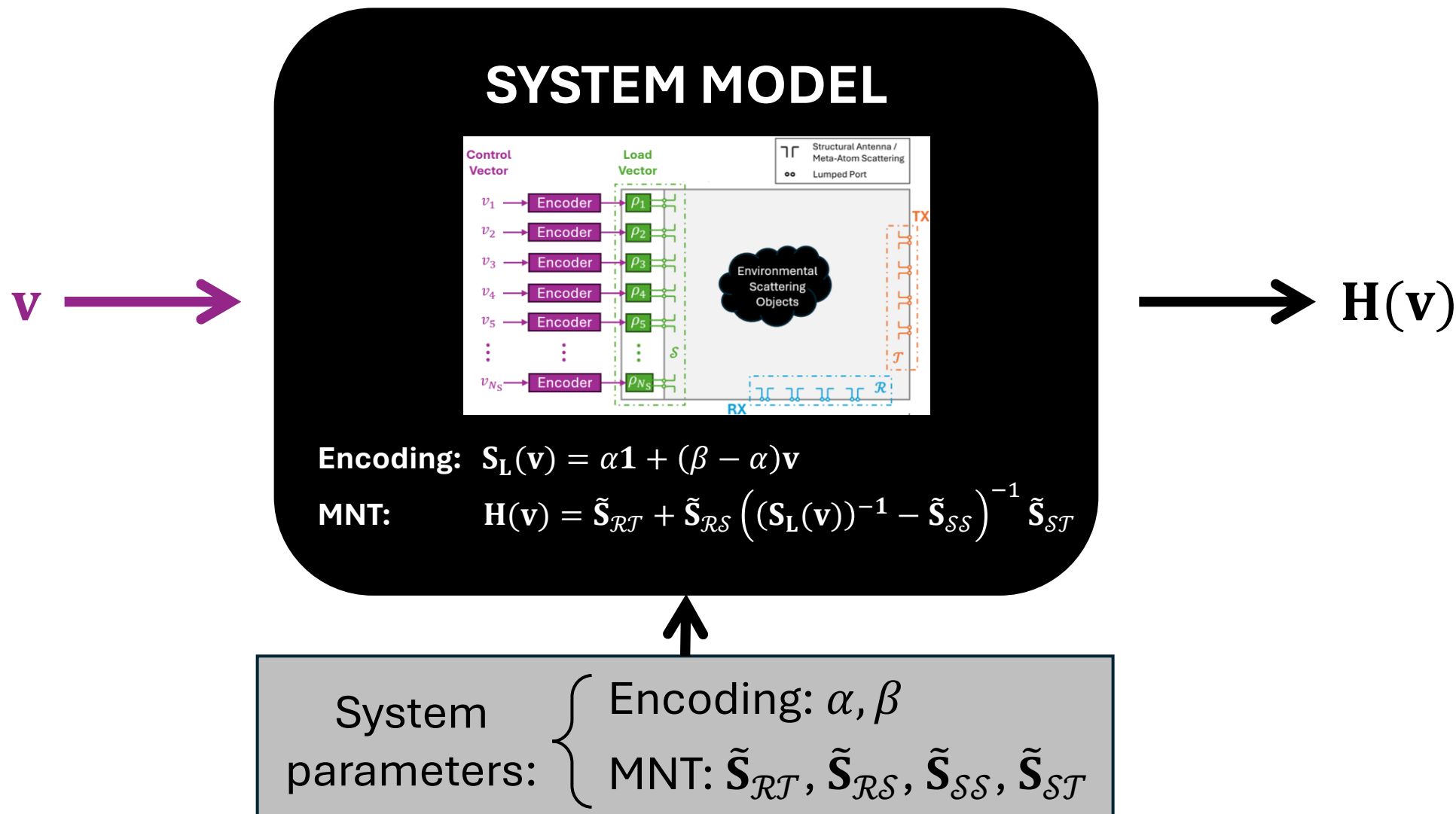
- continuous metallic boundaries
- extended dielectric layers
- structural scattering of RIS elements



2) Prototype-aware model calibration for reconfigurable microwave systems



2) Prototype-aware model calibration for reconfigurable microwave systems

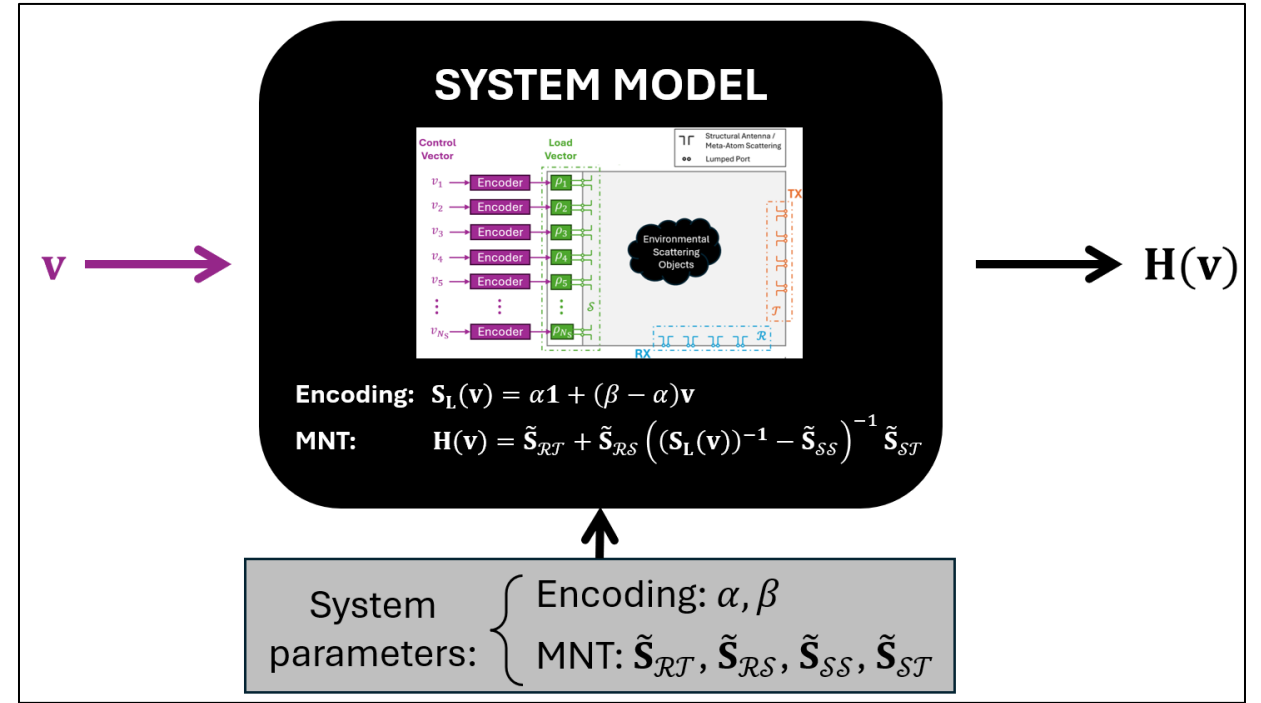


2) Prototype-aware model calibration for reconfigurable microwave systems

Based on indirect experimental measurements, the model parameters are usually not uniquely identifiable.

Nonetheless, we can estimate a set of proxy parameters that accurately maps any \mathbf{v} to the corresponding $\mathbf{H}(\mathbf{v})$.

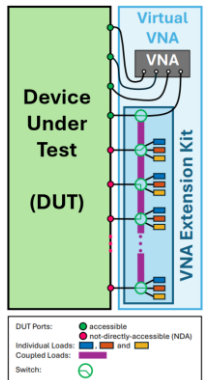
J. Sol, ..., P. del Hougne, *Nature Communications* (2024)
 P. del Hougne, *IEEE WCL* (2025), *IEEE TCOM* (2026), *IEEE WCL* (2026)
 J. Tapie, P. del Hougne, *arXiv: 2512.22607* (2025)



Eliminating all parameter ambiguities is impossible in most prototypes (\rightarrow Virtual VNA).

P. del Hougne, *IEEE TIM* (2025) [x3], *IEEE TAP* (2025) [x2]
 J. Tapie, P. del Hougne, *IEEE AWPL* (2025)

P. del Hougne, M. Di Renzo, A. Alù, T. J. Cui, Y. Eldar, N. Engheta, W. Hu, A. Ozcan, *HAL:05487878* (2026)



2) Prototype-aware model calibration for reconfigurable microwave systems

Diagonal-Similarity Gauge:

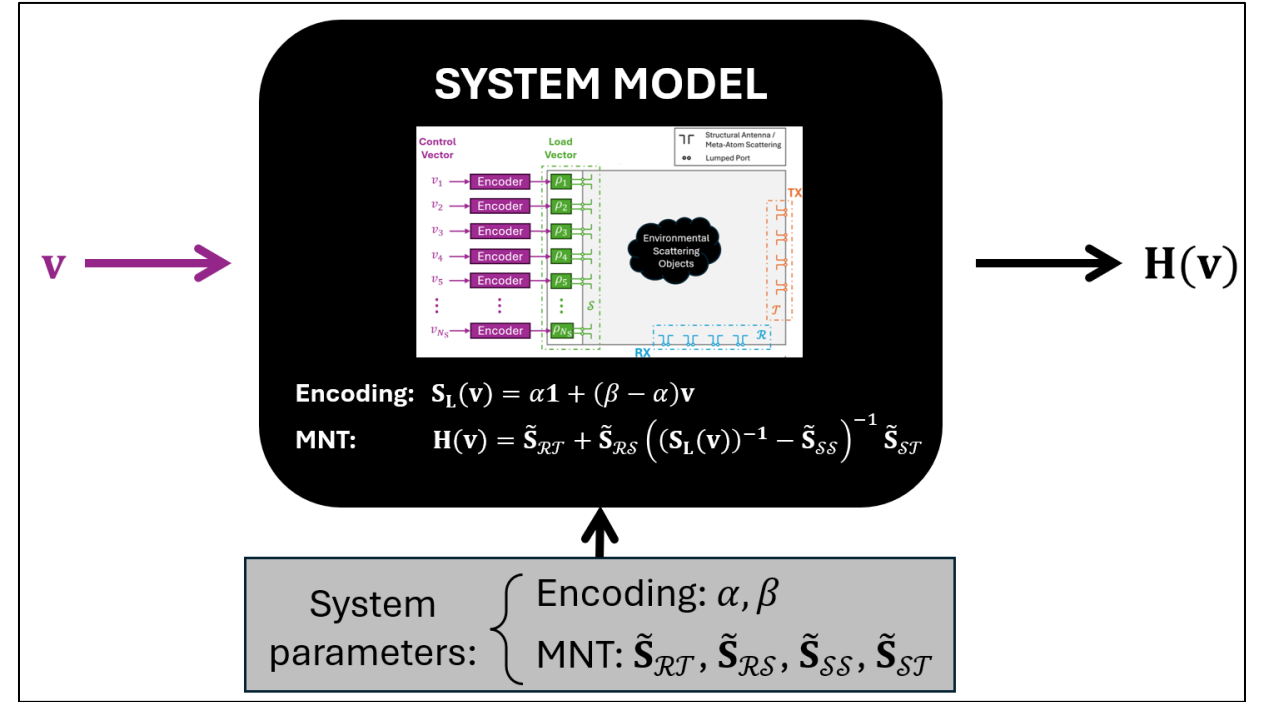
$\tilde{\mathbf{S}}_{\mathcal{RS}} \rightarrow \tilde{\mathbf{S}}_{\mathcal{RS}} \mathbf{D}^{-1}$, $\tilde{\mathbf{S}}_{\mathcal{SS}} \rightarrow \mathbf{D} \tilde{\mathbf{S}}_{\mathcal{SS}} \mathbf{D}^{-1}$, $\tilde{\mathbf{S}}_{\mathcal{ST}} \rightarrow \mathbf{D} \tilde{\mathbf{S}}_{\mathcal{ST}}$
for some invertible, complex-valued $\mathbf{D} = \text{diag}(\mathbf{d})$.

Complex-Scaling Gauge:

$\alpha \rightarrow c\alpha$, $\beta \rightarrow c\beta$, $\tilde{\mathbf{S}}_{\mathcal{RS}} \rightarrow \frac{1}{c} \tilde{\mathbf{S}}_{\mathcal{RS}}$, $\tilde{\mathbf{S}}_{\mathcal{SS}} \rightarrow \frac{1}{c} \tilde{\mathbf{S}}_{\mathcal{SS}}$
for some non-zero, complex-valued c .

Möbius Gauge:

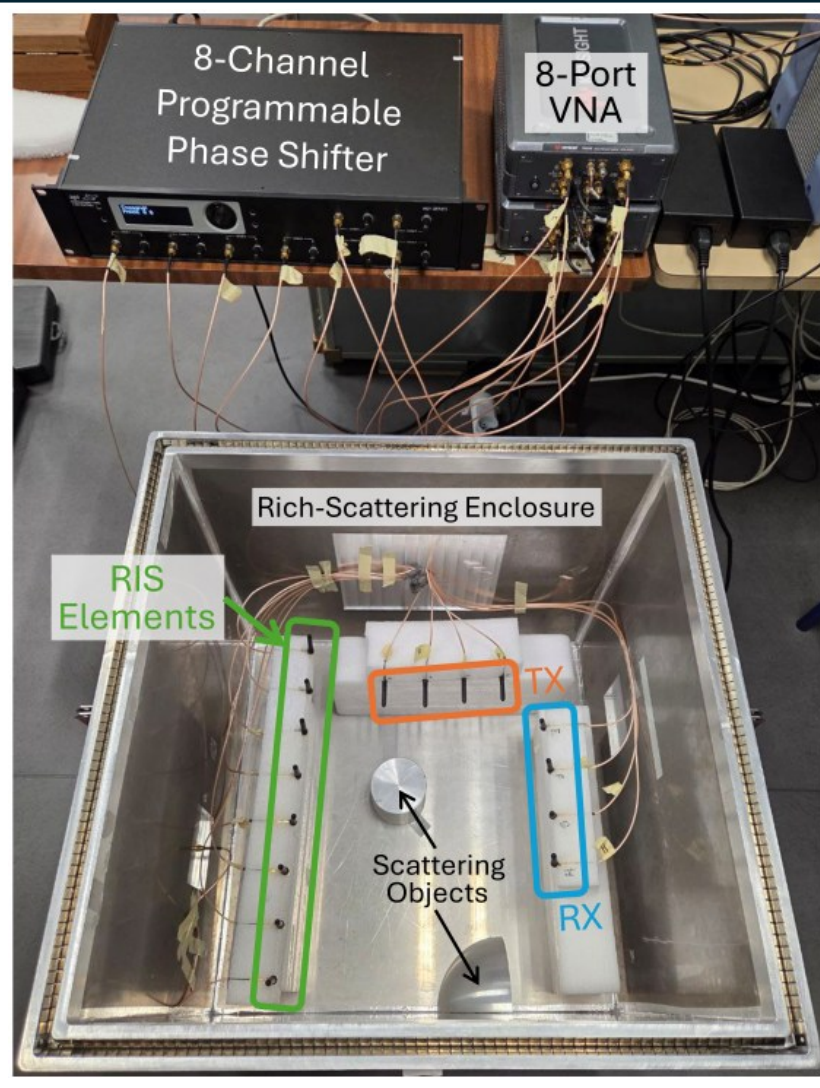
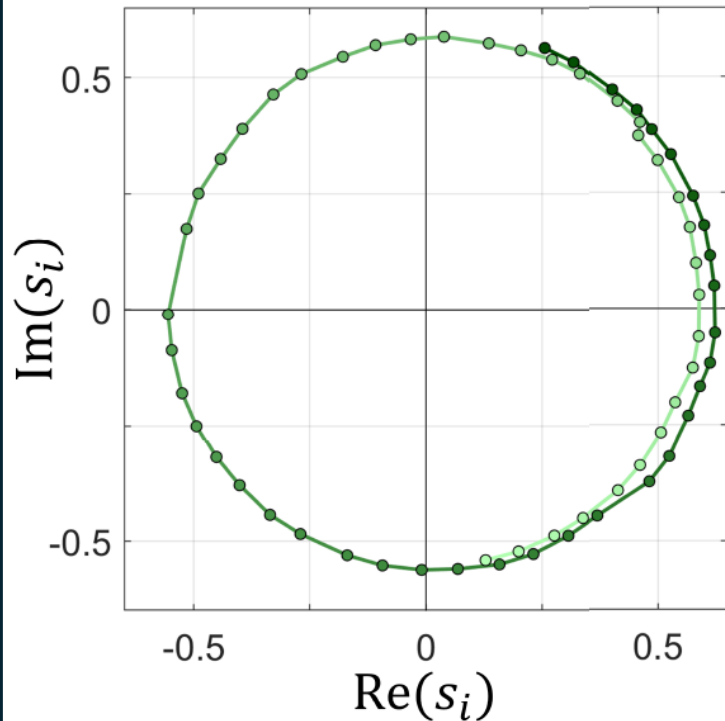
$\alpha \rightarrow M_m(\alpha)$, $\beta \rightarrow M_m(\beta)$, $\tilde{\mathbf{S}}_{\mathcal{RT}} \rightarrow \tilde{\mathbf{S}}_{\mathcal{RT}} + m \tilde{\mathbf{S}}_{\mathcal{RS}} \mathbf{F} \tilde{\mathbf{S}}_{\mathcal{ST}}$, $\tilde{\mathbf{S}}_{\mathcal{RS}} \rightarrow k \tilde{\mathbf{S}}_{\mathcal{RS}} \mathbf{F}$, $\tilde{\mathbf{S}}_{\mathcal{ST}} \rightarrow k \mathbf{F} \tilde{\mathbf{S}}_{\mathcal{ST}}$, $\tilde{\mathbf{S}}_{\mathcal{SS}} \rightarrow (\tilde{\mathbf{S}}_{\mathcal{SS}} - m^* \mathbf{I}) \mathbf{F}$
where $\mathbf{F} = (\mathbf{I} - m \tilde{\mathbf{S}}_{\mathcal{SS}})^{-1}$, $k = \sqrt{1 - |m|^2}$, and $M_m(r) = \frac{r-m}{1-m^*r}$,
for some non-zero, complex-valued m .



2) Prototype-aware model calibration for reconfigurable microwave systems

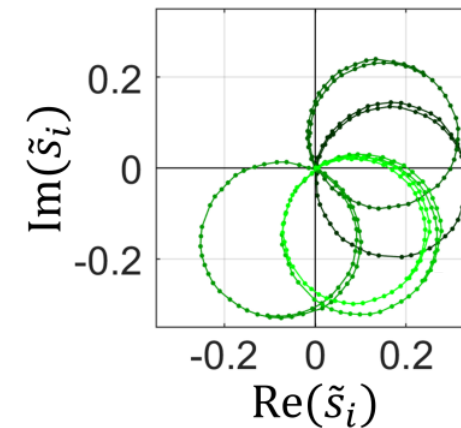
Modular 6-Bit RIS Prototype

Available load states s



Hybrid Parameter Estimation Approach

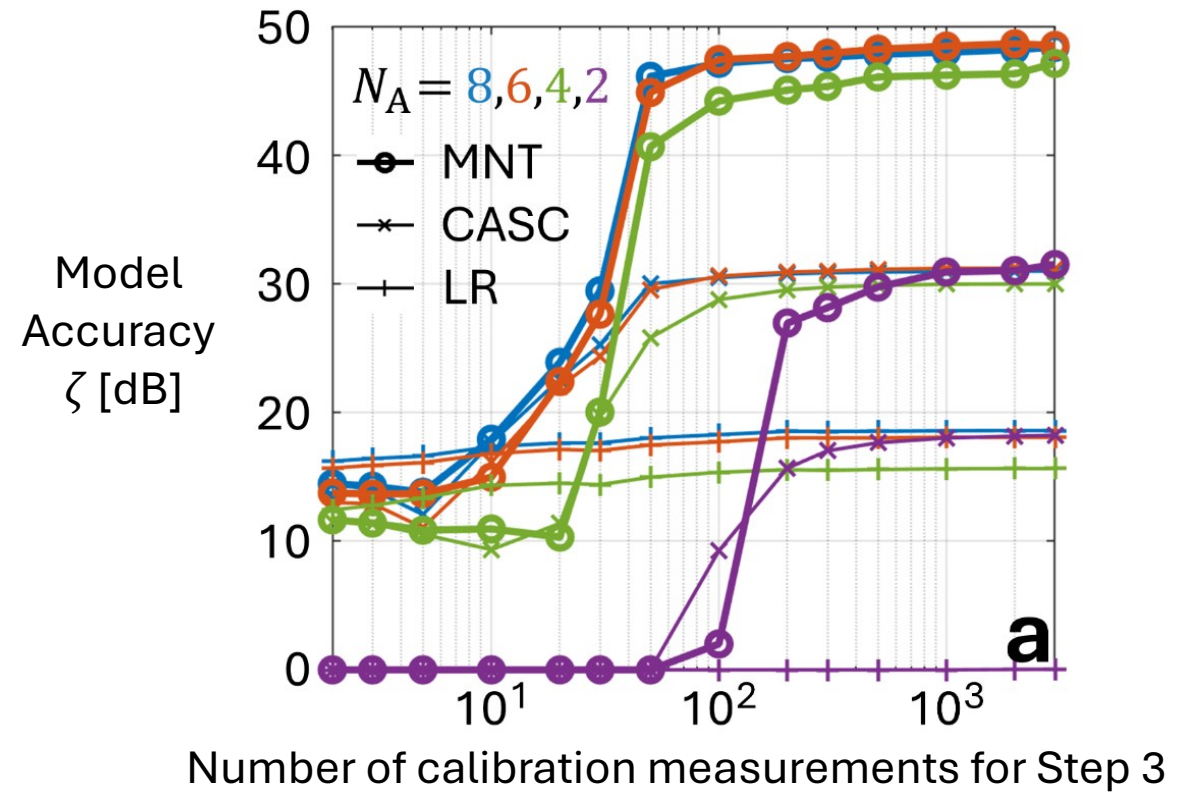
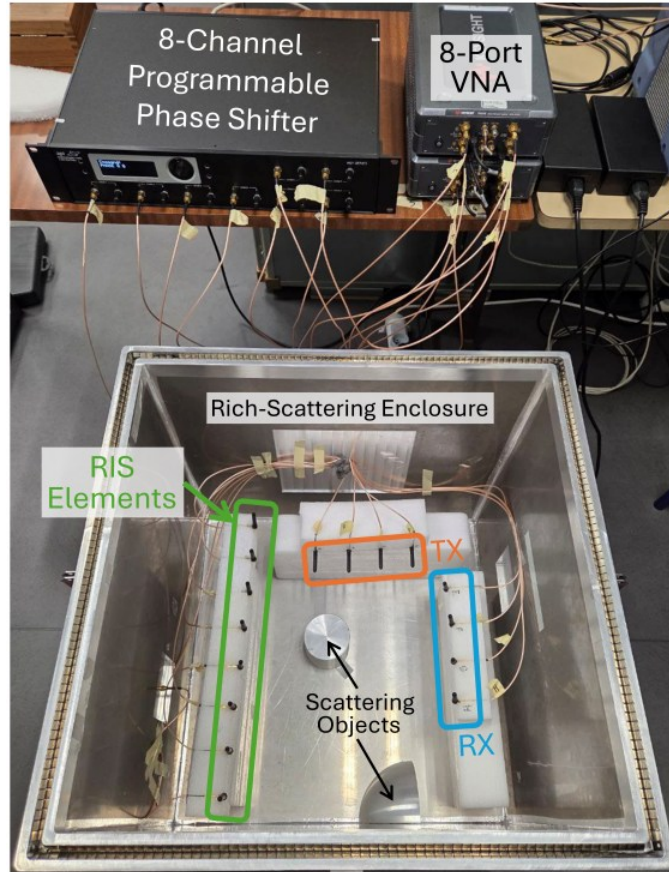
- 1) Fix $s_1 = 0$ and $\tilde{\mathbf{S}}_{\mathcal{RT}}$ to the value of \mathbf{H} in a reference RIS configuration.
- 2) Fix rows/columns of $\tilde{\mathbf{S}}_{\mathcal{RS}}$ and $\tilde{\mathbf{S}}_{\mathcal{ST}}$ up to scaling factors based on rank-1 changes of reference RIS configuration.
- 3) Fix remaining parameters via gradient descent based on measurements with random RIS configurations.



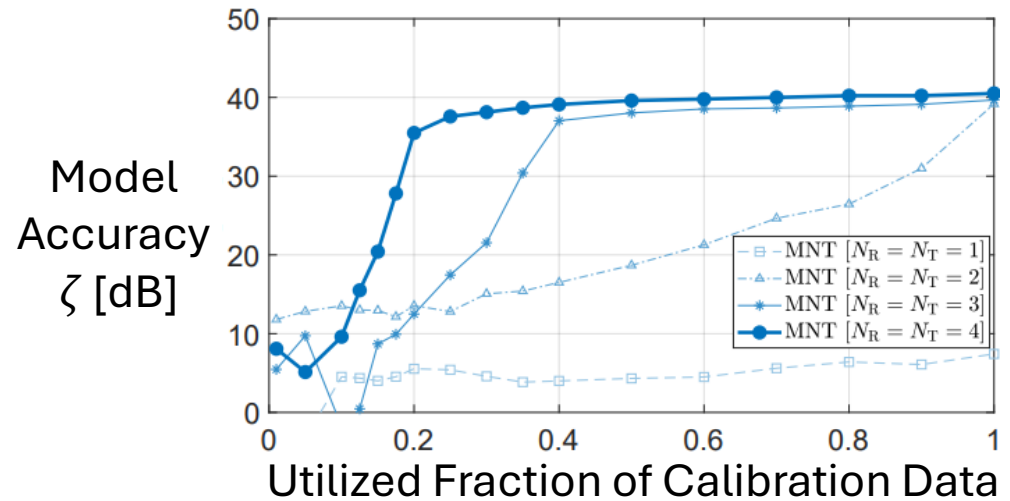
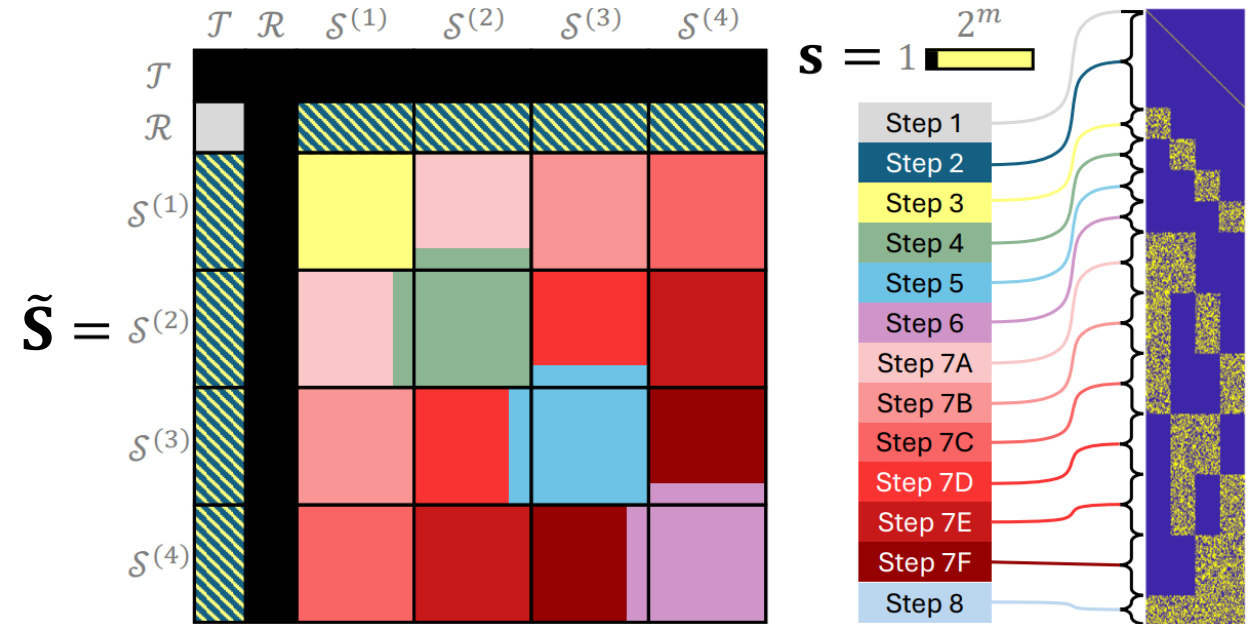
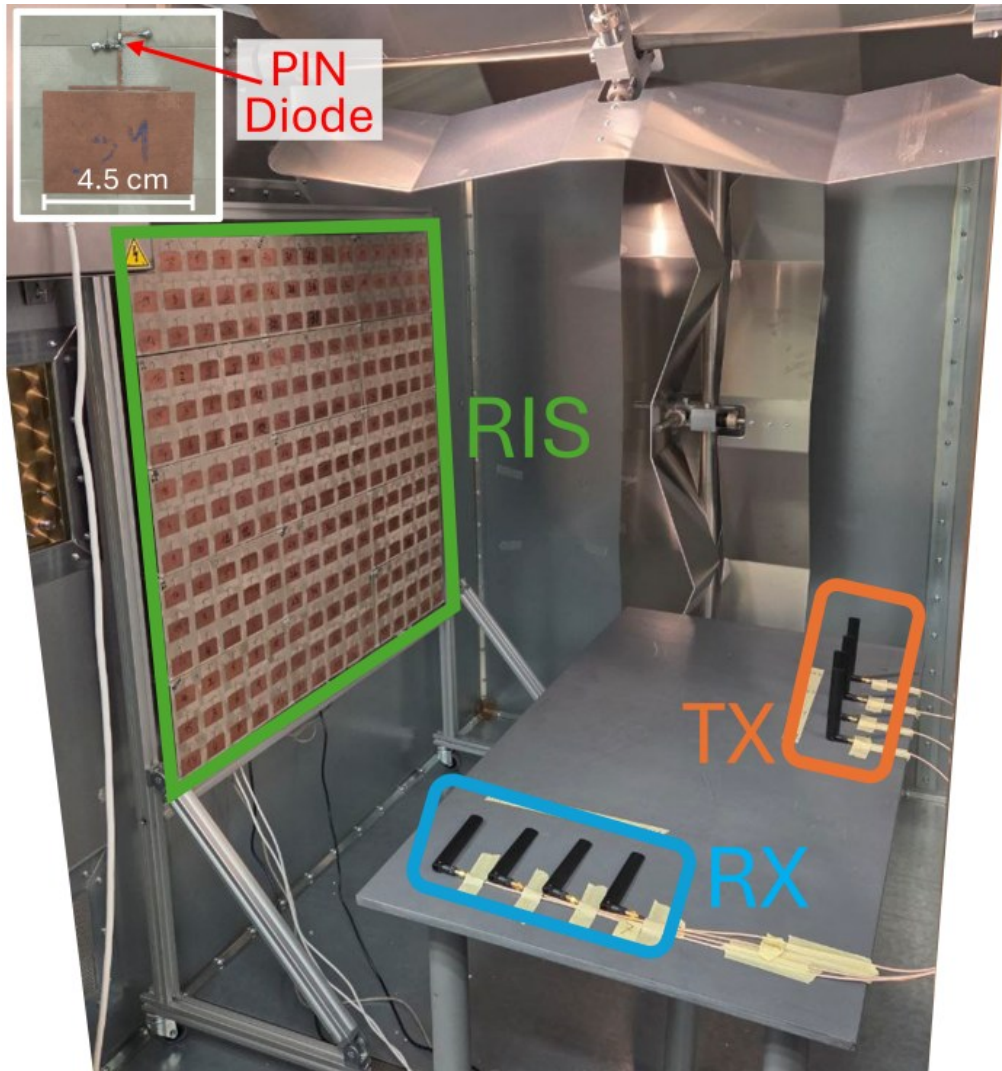
Estimates of \tilde{s}_i are Möbius transformations of ground-truth s_i .

2) Prototype-aware model calibration for reconfigurable microwave systems

Modular 6-Bit RIS Prototype



2) Prototype-aware model calibration for reconfigurable microwave systems



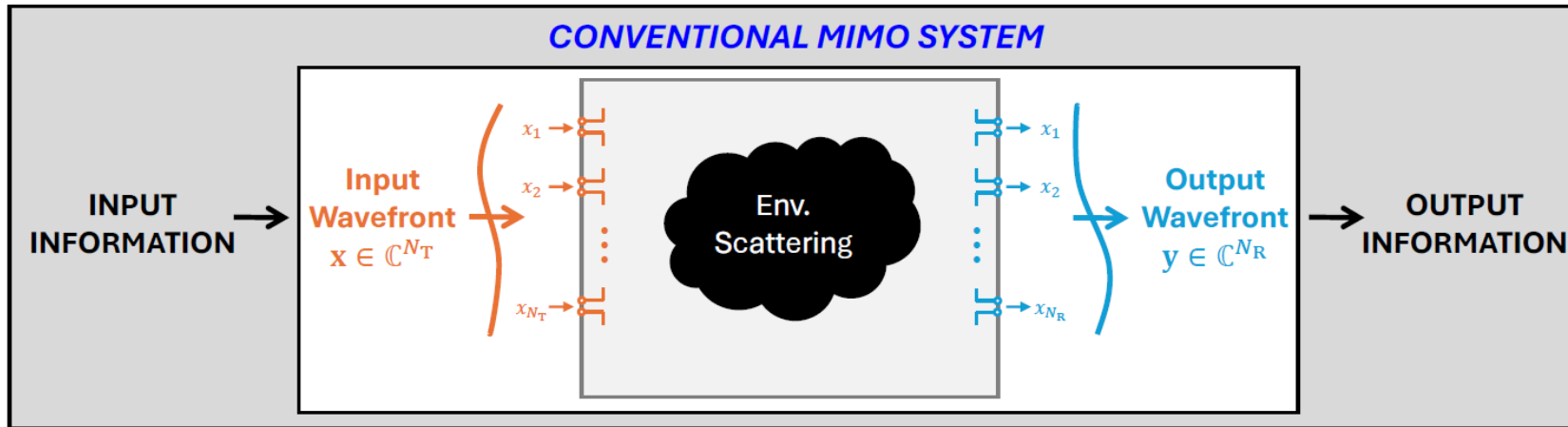
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3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on multiplexing gain in backscatter MIMO systems

Background on effective EM degrees of freedom (EEMDOFs) in conventional MIMO systems

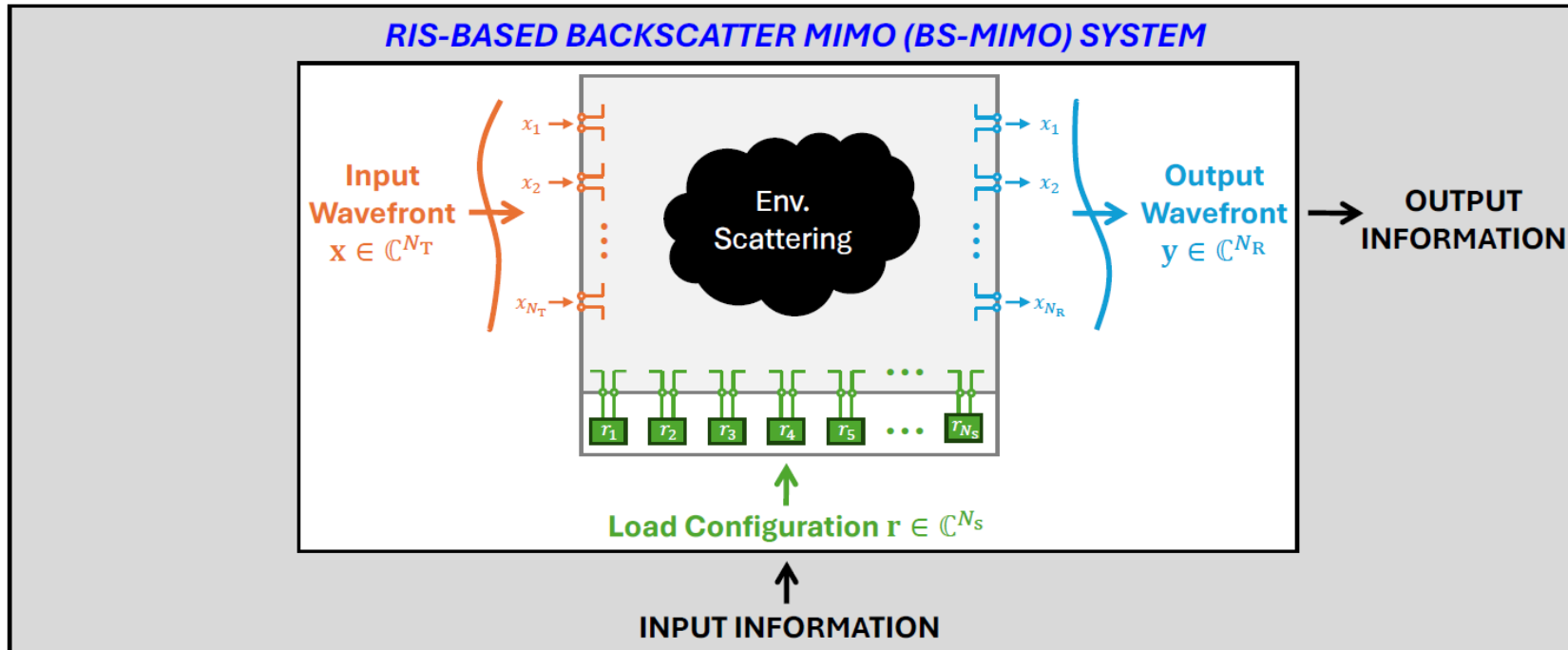


- Effective degrees of freedom quantify how many independent directions in the space of input parameters produce distinguishable changes of the outputs.
- In the high-SNR regime (where multi-eigenmode transmission is applicable), the number of EEMDOFs characterizes the multiplexing gain and the capacity scaling with SNR.
- The number of EEMDOFs is commonly defined as the number of significant singular values of the end-to-end channel matrix characterizing the MIMO system.

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on multiplexing gain in backscatter MIMO systems

Massive Backscatter MIMO system



- Mapping from input information to output information is non-linear due to mutual coupling.
- How can we define and evaluate BS-EEMDOFs?

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on multiplexing gain in backscatter MIMO systems

Generalized definition of effective degrees of freedom

- Input-to-output mapping $\boldsymbol{\theta}_{\text{out}} = f(\boldsymbol{\theta}_{\text{in}})$.
- Local affine approximation around reference point $\boldsymbol{\theta}_{\text{in},0}$:
 $\boldsymbol{\theta}_{\text{out}} \approx \boldsymbol{\theta}_{\text{out},0} + \mathbf{J}(\boldsymbol{\theta}_{\text{in},0})\delta\boldsymbol{\theta}_{\text{in}}$, where $\mathbf{J}(\boldsymbol{\theta}_{\text{in},0}) = \left. \frac{\partial \boldsymbol{\theta}_{\text{out}}}{\partial \boldsymbol{\theta}_{\text{in}}} \right|_{\boldsymbol{\theta}_{\text{in}}=\boldsymbol{\theta}_{\text{in},0}}$
- Jacobian's participation number quantifies the number of effective DoFs:

$$M(\boldsymbol{\theta}_{\text{in},0}) = PN(\mathbf{J}(\boldsymbol{\theta}_{\text{in},0})) = \frac{(\sum_{i=1}^{\tilde{N}} \sigma_i^2)^2}{\sum_{i=1}^{\tilde{N}} \sigma_i^4}$$

- Note: $M(\boldsymbol{\theta}_{\text{in},0})$ depends on $\boldsymbol{\theta}_{\text{in},0}$.

Sanity check on conventional MIMO:

- $\mathbf{y} = f(\mathbf{x}) = \mathbf{H}\mathbf{x}$
- $\mathbf{J}(\mathbf{x}_0) = \left. \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_0} = \mathbf{H} \quad \forall \mathbf{x}_0$
- $M(\mathbf{x}_0) = PN(\mathbf{H}) \quad \forall \mathbf{x}_0$

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on multiplexing gain in backscatter MIMO systems

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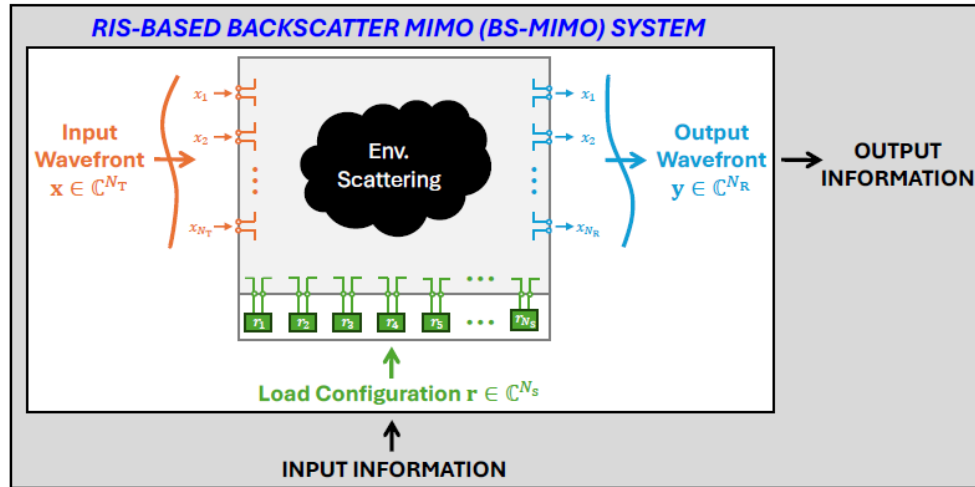
Application to Backscatter MIMO:

- $\mathbf{y} = f(\mathbf{r}, \mathbf{x}) = \left[\tilde{\mathbf{S}}_{\mathcal{R}\mathcal{T}} + \tilde{\mathbf{S}}_{\mathcal{R}\mathcal{S}} (\mathbf{I}_{N_S} - \boldsymbol{\Phi}(\mathbf{r}) \tilde{\mathbf{S}}_{\mathcal{S}\mathcal{S}})^{-1} \boldsymbol{\Phi}(\mathbf{r}) \tilde{\mathbf{S}}_{\mathcal{S}\mathcal{T}} \right] \mathbf{x}$, where $\boldsymbol{\Phi}(\mathbf{r}) = \text{diag}(\mathbf{r})$.
- $\mathbf{J}(\mathbf{r}_0, \mathbf{x}) = \left. \frac{\partial \mathbf{y}}{\partial \mathbf{r}} \right|_{\mathbf{r}=\mathbf{r}_0} = \tilde{\mathbf{S}}_{\mathcal{R}\mathcal{S}} \mathbf{B}(\mathbf{r}_0, \mathbf{x})$.
- $M(\mathbf{r}_0, \mathbf{x}) = PN(\mathbf{J}(\mathbf{r}_0, \mathbf{x}))$
- Note: BS-EEMDOFs depend on \mathbf{r}_0 and \mathbf{x} .
- If $\tilde{\mathbf{S}}_{\mathcal{S}\mathcal{S}} = \mathbf{0}$ (implying no mutual coupling), $\mathbf{J}(\mathbf{r}_0, \mathbf{x}) = \tilde{\mathbf{S}}_{\mathcal{R}\mathcal{S}} \text{diag}(\tilde{\mathbf{S}}_{\mathcal{S}\mathcal{T}} \mathbf{x}) \forall \mathbf{r}_0$.

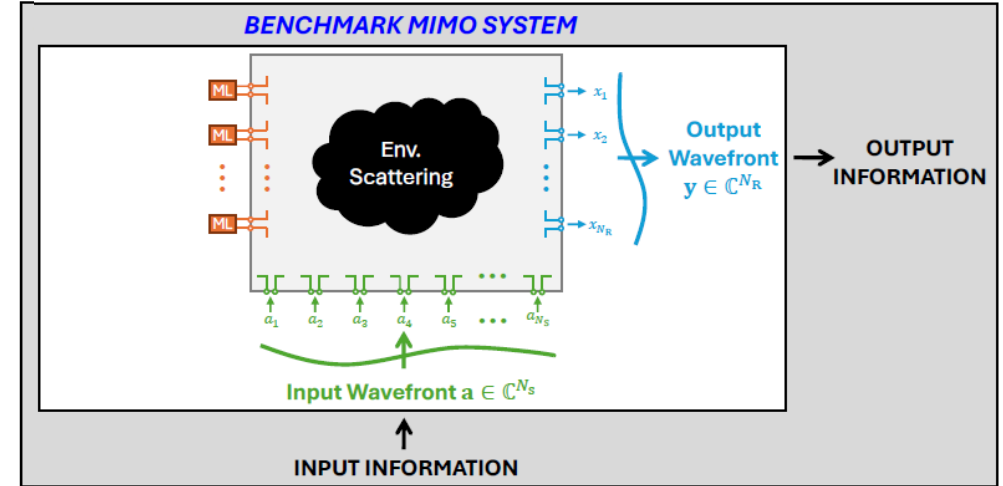
$$\begin{aligned} \mathbf{B}(\mathbf{r}_0, \mathbf{x}) &= \mathbf{G}(\mathbf{r}_0) \text{diag}(\mathbf{W}(\mathbf{r}_0) \mathbf{x}) \\ \mathbf{W}(\mathbf{r}) &= \tilde{\mathbf{S}}_{\mathcal{S}\mathcal{S}} \mathbf{G}(\mathbf{r}) \boldsymbol{\Phi}(\mathbf{r}) \tilde{\mathbf{S}}_{\mathcal{S}\mathcal{T}} + \tilde{\mathbf{S}}_{\mathcal{S}\mathcal{T}} \\ \mathbf{G}(\mathbf{r}) &= (\mathbf{I}_{N_S} - \boldsymbol{\Phi}(\mathbf{r}) \tilde{\mathbf{S}}_{\mathcal{S}\mathcal{S}})^{-1} \end{aligned}$$

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on multiplexing gain in backscatter MIMO systems



$$\text{Local mapping: } \mathbf{J}(\mathbf{r}_0, \mathbf{x}) = \tilde{\mathbf{S}}_{RS} \mathbf{B}(\mathbf{r}_0, \mathbf{x})$$

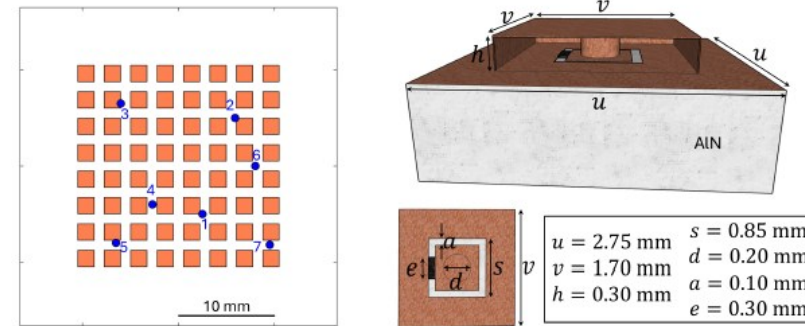
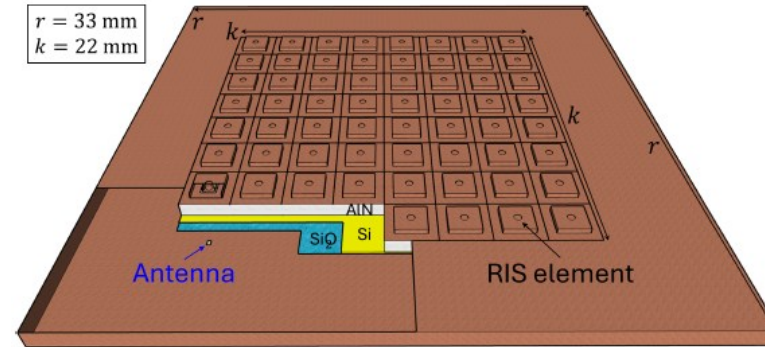
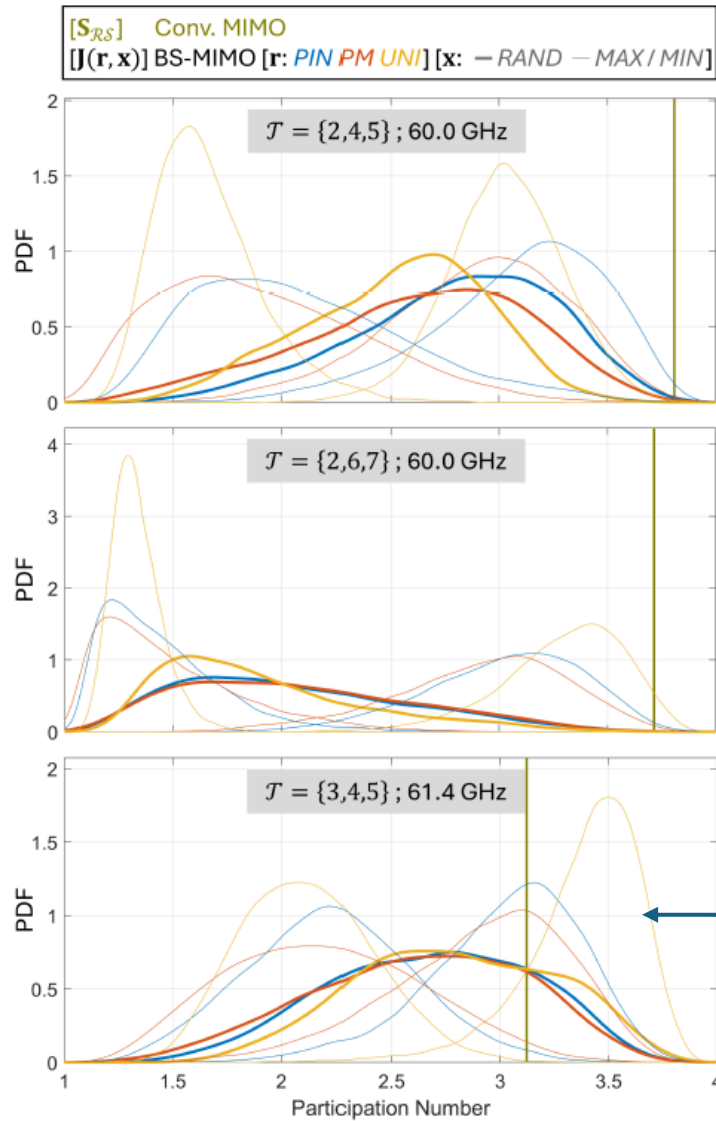


$$\text{Mapping: } \tilde{\mathbf{S}}_{RS}$$

- Modes associated with BS-EEMDOFs lie in the same column space as modes associated with EEMDOFs.
- The number of BS-EEMDOFs generally differs from the number of EEMDOFs.
- The number of BS-EEMDOFs depends on \mathbf{r}_0 and \mathbf{x} .
- The number of BS-EEMDOFs is generally a random variable (unless MC is negligible and illumination is fixed).
- The distribution of the number of BS-EEMDOFs can be optimized by controlling the illumination.

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on multiplexing gain in backscatter MIMO systems



BS-EEMDOFs clearly exceed EEMDOFs.

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on multiplexing gain in backscatter MIMO systems

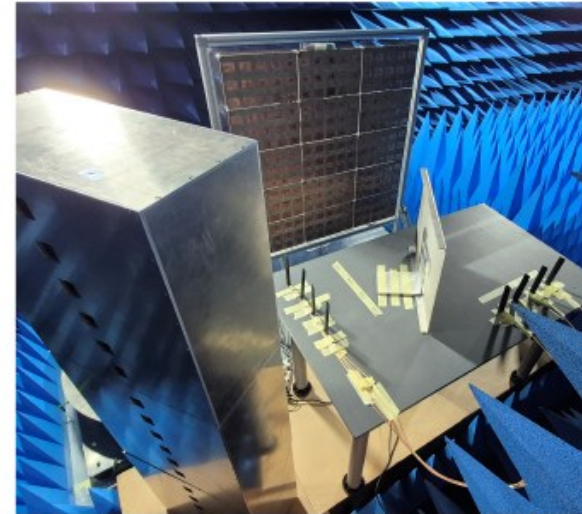
Rich Env. Scattering



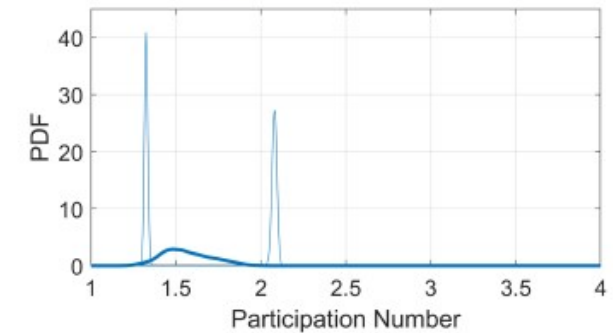
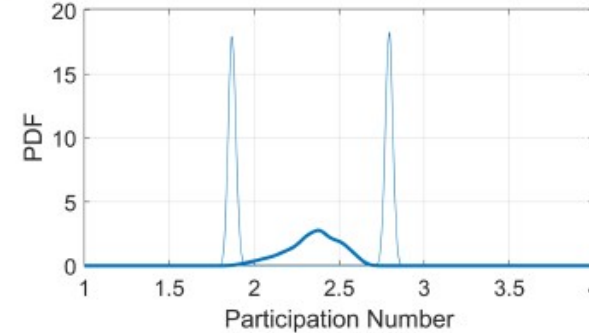
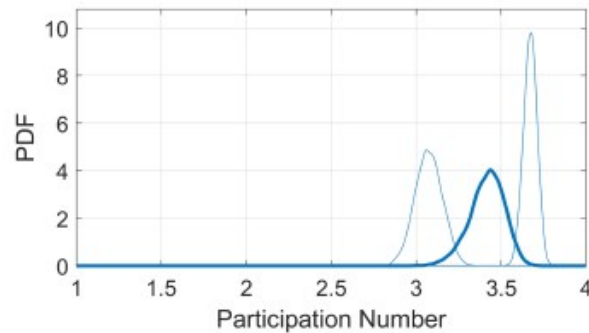
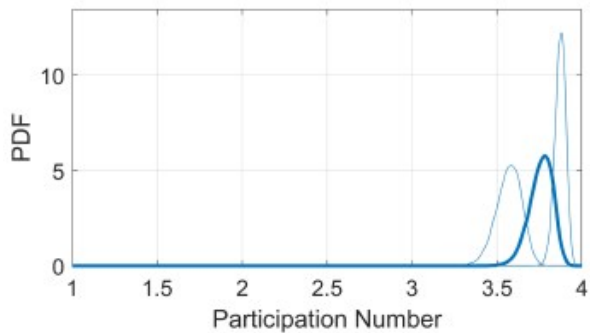
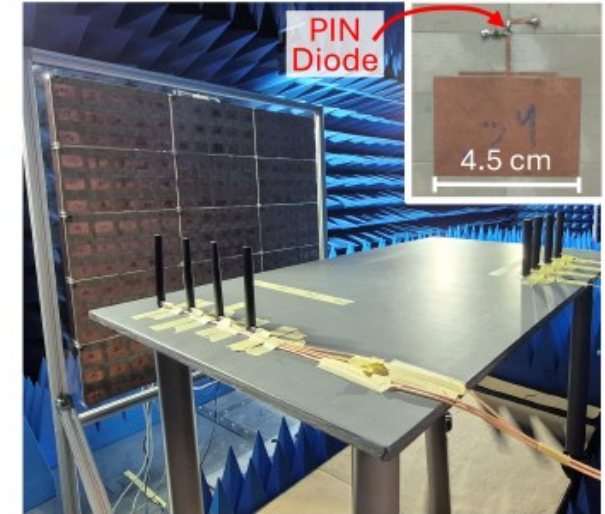
Attenuated Rich Env. Scattering



Light Env. Scattering



No Env. Scattering



Outline

- 1) **Multiport-network model formulation** for reconfigurable microwave systems
- 2) **Prototype-aware model calibration** for reconfigurable microwave systems
- 3) **Toward a prototype-aware EM information theory** for reconfigurable microwave systems
 - Bounds on multiplexing gain in backscatter MIMO systems
 - Bounds on information transfer via RIS-parametrized SISO channel
 - Bounds on MIMO operator synthesis

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on information transfer via RIS-parametrized SISO channel

Shannon's theorem for achievable reliable information rate on a noisy SISO channel:

$$C = \log_2 \left(1 + \frac{P_T}{\sigma^2} |h|^2 \right)$$

Now, we have **deterministically programmable channels**:

$$C(\mathbf{v}) = \log_2 \left(1 + \frac{P_T}{\sigma^2} |h(\mathbf{v})|^2 \right)$$

→ What is the bound on the achievable information transfer in a programmable channel subject to the constraints of electromagnetics and a real-world feasibility set?

In the SISO case, channel gain maps monotonously to Shannon capacity...

→ What is the bound on the channel gain in a programmable channel subject to the constraints of electromagnetics and a real-world feasibility set?

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on information transfer via RIS-parametrized SISO channel

Objective: SISO channel gain maximization.

$$\begin{aligned} h(\mathbf{r}) &= h_0 + \mathbf{a}^\top (\mathbf{I}_{N_S} - \Phi(\mathbf{r}) \mathbf{\Gamma})^{-1} \Phi(\mathbf{r}) \mathbf{b} \\ \Phi(\mathbf{r}) &= \text{diag}(\mathbf{r}) \\ \mathbf{r}(\mathbf{v}) &= \alpha \mathbf{1} + (\beta - \alpha) \mathbf{v} \end{aligned}$$

How can we bound the achievable SISO channel gain $|h|^2$ for a given experimental system?

1) Norm-Inequality Bound

$$|h(\mathbf{r})|^2 \leq B_{\text{NI}} \triangleq \left(|h_0| + \|\mathbf{a}\|_2 \frac{\gamma}{1 - \gamma \|\mathbf{\Gamma}\|_2} \|\mathbf{b}\|_2 \right)^2$$

This bound is sensitive to model parameter ambiguities.

→ We can tighten this bound by optimizing over model parameter ambiguities.

$$\begin{aligned} B_{\text{NIO}}(\boldsymbol{\theta}) &\triangleq \min_{\mathbf{d}, m} B_{\text{NI}}(\hat{g}(\boldsymbol{\theta}; [\mathbf{d}^\top, c_0, m]^\top)) \\ \text{s.t. } &\tilde{\gamma} \|\tilde{\mathbf{\Gamma}}\|_2 < 1, \quad d_i \neq 0 \quad \forall i, \quad \text{all inverses in } \hat{g}(\cdot) \text{ exist} \end{aligned}$$

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on information transfer via RIS-parametrized SISO channel

Objective: SISO channel gain maximization.

$$\begin{aligned}h(\mathbf{r}) &= h_0 + \mathbf{a}^\top (\mathbf{I}_{N_S} - \Phi(\mathbf{r}) \Gamma)^{-1} \Phi(\mathbf{r}) \mathbf{b} \\ \Phi(\mathbf{r}) &= \text{diag}(\mathbf{r}) \\ \mathbf{r}(\mathbf{v}) &= \alpha \mathbf{1} + (\beta - \alpha) \mathbf{v}\end{aligned}$$

How can we bound the achievable SISO channel gain $|h|^2$ for a given experimental system?

2) Idealized-BD-RIS Bound

If $|\alpha| = |\beta| = 1$ and $\|\Gamma\|_2 < 1$,

then the closed-form global solution for the case in which Φ can be any unitary matrix is a bound.

Using the auxiliary variable $\mathbf{x} \triangleq (\mathbf{I}_{N_S} - \Phi \Gamma)^{-1} \Phi \mathbf{b}$,

the problem can be reframed as maximizing an affine functional over an ellipsoid, for which a closed-form solution via Cauchy-Schwarz alignment exists.

See also:

Z. Wu, M. Nerini, and B. Clerckx, *arXiv:2510.12366* (2025)

Being a global solution, this bound B_{IBD} is by construction insensitive to model parameter ambiguities.

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on information transfer via RIS-parametrized SISO channel

Objective: SISO channel gain maximization.

$$\begin{aligned} h(\mathbf{r}) &= h_0 + \mathbf{a}^\top (\mathbf{I}_{N_S} - \Phi(\mathbf{r}) \Gamma)^{-1} \Phi(\mathbf{r}) \mathbf{b} \\ \Phi(\mathbf{r}) &= \text{diag}(\mathbf{r}) \\ \mathbf{r}(\mathbf{v}) &= \alpha \mathbf{1} + (\beta - \alpha) \mathbf{v} \end{aligned}$$

How can we bound the achievable SISO channel gain $|h|^2$ for a given experimental system?

3) Semidefinite Relaxation Bound

Introducing the auxiliary variable $\mathbf{x} \triangleq (\mathbf{I}_{N_S} - \Phi \Gamma)^{-1} \Phi \mathbf{b}$,

we can formulate our problem as QCQP:

By introducing $\mathbf{X} = \mathbf{x}\mathbf{x}^\dagger$, a semidefinite relaxation yields an SDP that can be solved with standard convex solvers:

$$\begin{aligned} \max_{\mathbf{x} \in \mathbb{C}^{N_S}} \quad & \mathbf{x}^\dagger \mathbf{R}_0 \mathbf{x} + 2 \Re\{\mathbf{q}_0^\top \mathbf{x}\} + t_0 \\ \text{s.t.} \quad & \mathbf{x}^\dagger \mathbf{R}_i \mathbf{x} + \mathbf{x}^\dagger \mathbf{q}_{1,i} + \mathbf{q}_{2,i}^\top \mathbf{x} + t_i = 0, \quad i = 1, \dots, N_S. \end{aligned}$$

$$\begin{aligned} \max_{\mathbf{x}, \mathbf{X}} \quad & \text{tr}(\mathbf{R}_0 \mathbf{X}) + 2 \Re\{\mathbf{q}_0^\top \mathbf{x}\} + t_0 \\ \text{s.t.} \quad & \text{tr}(\mathbf{R}_i \mathbf{X}) + \mathbf{x}^\dagger \mathbf{q}_{1,i} + \mathbf{q}_{2,i}^\top \mathbf{x} + t_i = 0, \quad i = 1, \dots, N_S, \\ & \begin{bmatrix} \mathbf{X} & \mathbf{x} \\ \mathbf{x}^\dagger & 1 \end{bmatrix} \succeq \mathbf{0}. \end{aligned}$$

Any feasible point for QCQP is also feasible for SDP, hence

$B_{\text{SDR}} \triangleq \text{tr}(\mathbf{R}_0 \check{\mathbf{X}}) + 2 \Re\{\mathbf{q}_0^\top \check{\mathbf{x}}\} + t_0$ (which we can show to be ambiguity-insensitive).

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on information transfer via RIS-parametrized SISO channel

$$h(\mathbf{r}) = h_0 + \mathbf{a}^\top (\mathbf{I}_{N_S} - \Phi(\mathbf{r}) \Gamma)^{-1} \Phi(\mathbf{r}) \mathbf{b}$$

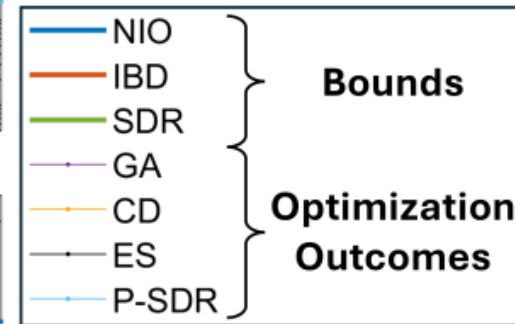
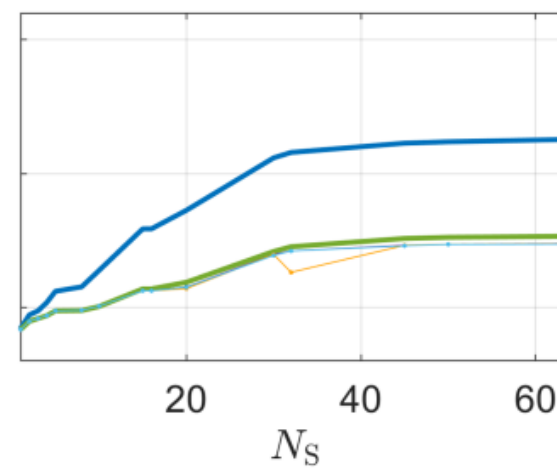
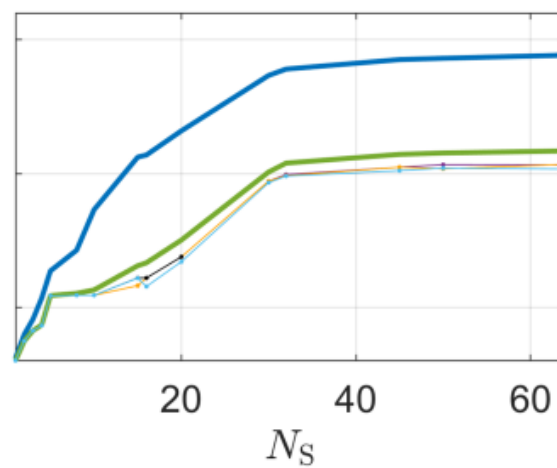
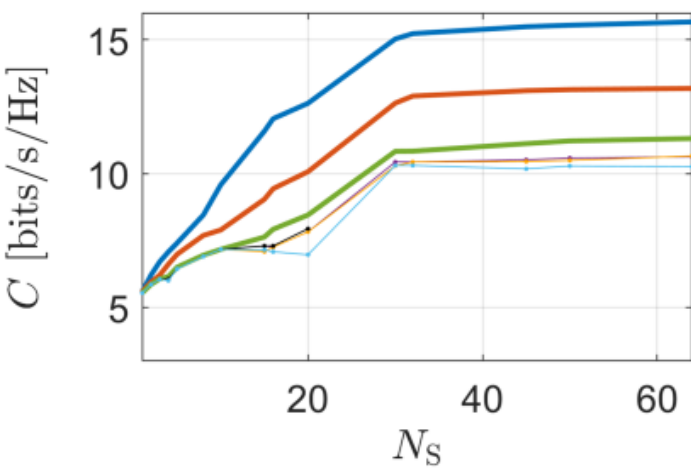
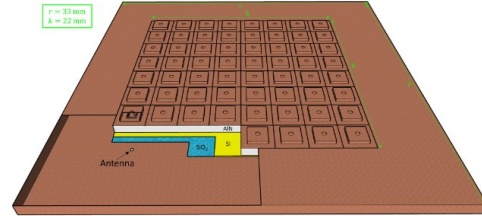
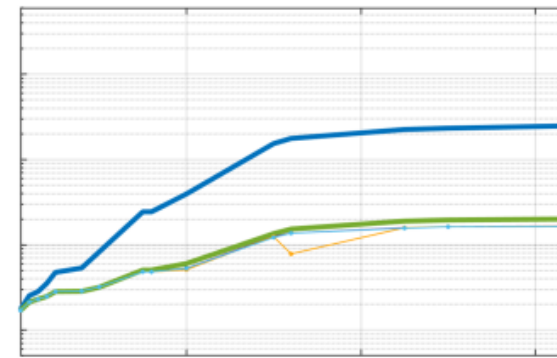
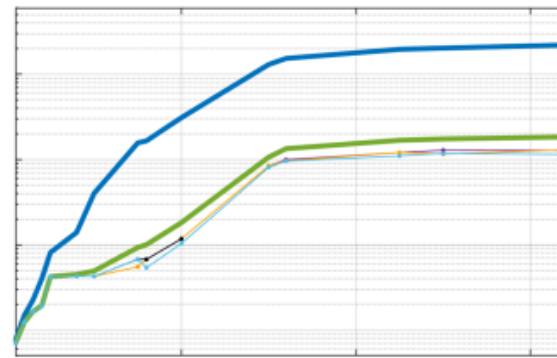
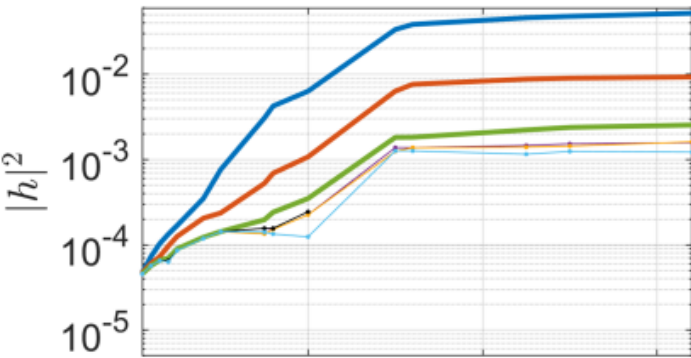
$$\Phi(\mathbf{r}) = \text{diag}(\mathbf{r})$$

$$\mathbf{r}(\mathbf{v}) = \alpha \mathbf{1} + (\beta - \alpha) \mathbf{v}$$

$\alpha = -1,$
 $\beta = 1$

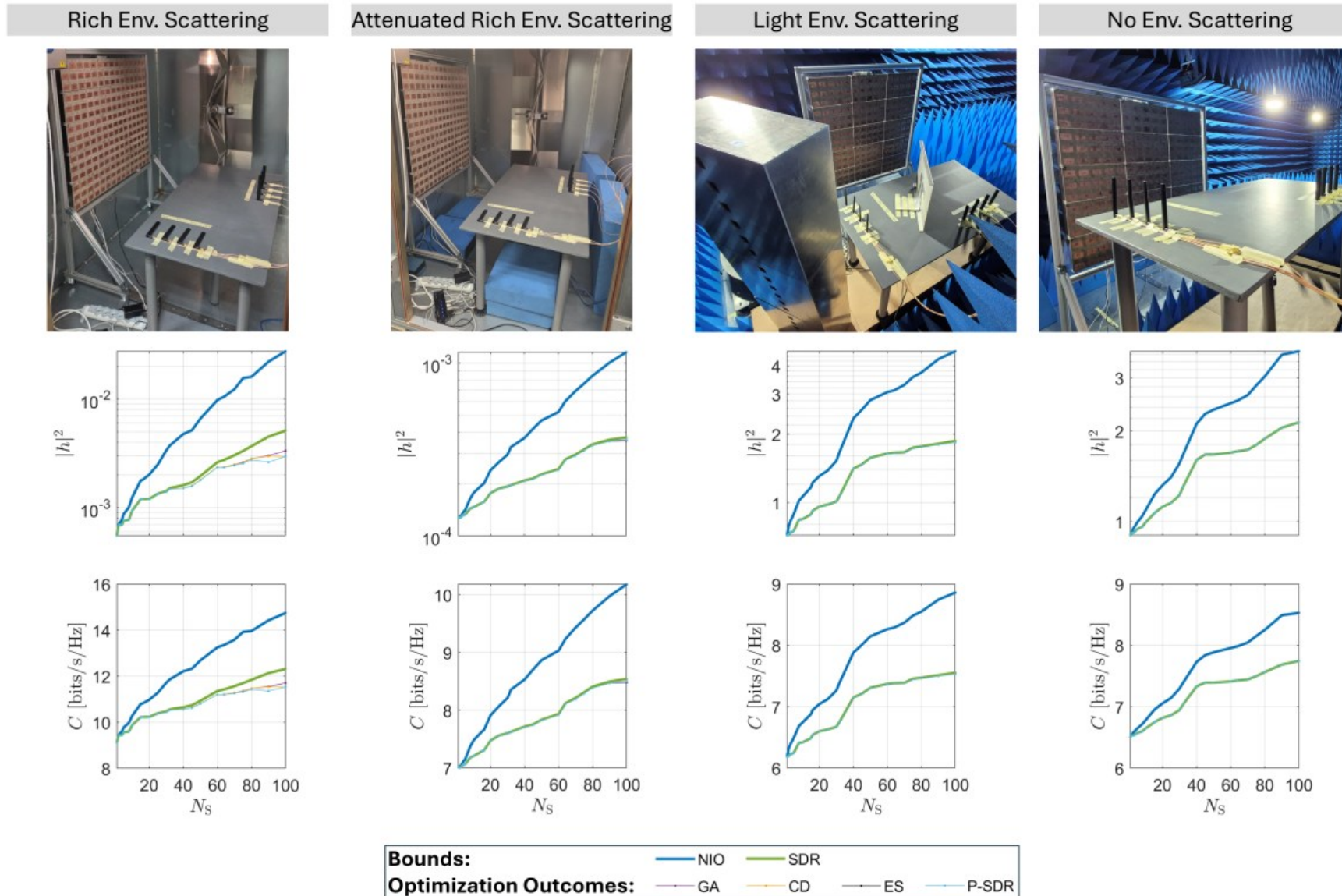
$\alpha = 0,$
 $\beta = 1$

$\alpha = 0.64 - 0.77j,$
 $\beta = -0.81$



3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on information transfer via RIS-parametrized SISO channel



Outline

- 1) **Multiport-network model formulation** for reconfigurable microwave systems
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 - Bounds on MIMO operator synthesis

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on MIMO operator synthesis

How well can a reconfigurable MIMO system's transfer function approximate a desired linear operator?

Relevance:

- hybrid-MIMO analog combining
- computational meta-imaging with illuminations based on principal scene components
- programmable wave-domain signal processing

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on MIMO aggregate channel gain

Let us first consider a simpler intermediate goal:

MIMO aggregate channel gain maximization.

$$\begin{aligned} \mathbf{H}(\mathbf{r}) &= \mathbf{H}_0 + \mathbf{A} (\mathbf{I}_{N_S} - \Phi(\mathbf{r}) \Gamma)^{-1} \Phi(\mathbf{r}) \mathbf{B} \\ \Phi(\mathbf{r}) &= \text{diag}(\mathbf{r}) \\ \mathbf{r}(\mathbf{v}) &= \alpha \mathbf{1} + (\beta - \alpha) \mathbf{v} \end{aligned}$$

Upon introducing the auxiliary variable $\mathbf{X} \triangleq (\mathbf{I}_{N_S} - \Phi(\mathbf{r}) \Gamma)^{-1} \Phi(\mathbf{r}) \mathbf{B}$ and vectorizing it with $\mathbf{y} \triangleq \text{vec}(\mathbf{X})$, we can formulate our problem as QCQP:

$\begin{aligned} \max_{\mathbf{y} \in \mathbb{C}^{N_S N_T}} \quad & \mathbf{y}^\dagger \mathbf{R}_0 \mathbf{y} + 2 \Re \{ \mathbf{q}_0^\dagger \mathbf{y} \} + \tau_0 \\ \text{s.t.} \quad & \mathbf{y}^\dagger \mathbf{R}_{s,t} \mathbf{y} + \mathbf{y}^\dagger \mathbf{p}_{s,t} + \mathbf{q}_{s,t}^\dagger \mathbf{y} + \tau_{s,t} = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{1, \dots, N_T\}, \\ & \mathbf{y}^\dagger \mathbf{R}_{s,t}^{(i)} \mathbf{y} + \mathbf{y}^\dagger \mathbf{p}_{s,t}^{(i)} + (\mathbf{q}_{s,t}^{(i)})^\dagger \mathbf{y} + \tau_{s,t}^{(i)} = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{2, \dots, N_T\}, \\ & \quad \forall i \in \{1, 2\}. \end{aligned}$	<p>← Quadratic objective</p> <p>← Quadratic binary-programmability constraint</p> <p>← Quadratic repetition constraint</p>
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See also:

H. Shim et al., *Nanophotonics* (2024)
A. Salmi et al., *IEEE TAP* (2025)

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on MIMO aggregate channel gain

Let us first consider a simpler intermediate goal:

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Upon introducing the auxiliary variable $\mathbf{X} \triangleq (\mathbf{I}_{N_S} - \Phi(\mathbf{r}) \Gamma)^{-1} \Phi(\mathbf{r}) \mathbf{B}$ and vectorizing it with $\mathbf{y} \triangleq \text{vec}(\mathbf{X})$, we can formulate our problem as QCQP:

$$\begin{aligned} \max_{\mathbf{y} \in \mathbb{C}^{N_S N_T}} \quad & \mathbf{y}^\dagger \mathbf{R}_0 \mathbf{y} + 2 \Re \{ \mathbf{q}_0^\dagger \mathbf{y} \} + \tau_0 \\ \text{s.t.} \quad & \mathbf{y}^\dagger \mathbf{R}_{s,t} \mathbf{y} + \mathbf{y}^\dagger \mathbf{p}_{s,t} + \mathbf{q}_{s,t}^\dagger \mathbf{y} + \tau_{s,t} = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{1, \dots, N_T\}, \\ & \mathbf{y}^\dagger \mathbf{R}_{s,t}^{(i)} \mathbf{y} + \mathbf{y}^\dagger \mathbf{p}_{s,t}^{(i)} + (\mathbf{q}_{s,t}^{(i)})^\dagger \mathbf{y} + \tau_{s,t}^{(i)} = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{2, \dots, N_T\}, \\ & \quad \forall i \in \{1, 2\}. \end{aligned}$$

Introduce $\mathbf{Y} = \mathbf{y} \mathbf{y}^\dagger$
and apply SDR

$$\begin{aligned} \max_{\mathbf{y}, \mathbf{Y}} \quad & \text{tr}(\mathbf{R}_0 \mathbf{Y}) + 2 \Re \{ \mathbf{q}_0^\dagger \mathbf{y} \} + \tau_0 \\ \text{s.t.} \quad & \text{tr}(\mathbf{R}_{s,t} \mathbf{Y}) + \mathbf{y}^\dagger \mathbf{p}_{s,t} + \mathbf{q}_{s,t}^\dagger \mathbf{y} + \tau_{s,t} = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{1, \dots, N_T\}, \\ & \text{tr}(\mathbf{R}_{s,t}^{(i)} \mathbf{Y}) + \mathbf{y}^\dagger \mathbf{p}_{s,t}^{(i)} + (\mathbf{q}_{s,t}^{(i)})^\dagger \mathbf{y} + \tau_{s,t}^{(i)} = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{2, \dots, N_T\}, \\ & \quad \forall i \in \{1, 2\}, \\ & \begin{bmatrix} \mathbf{Y} & \mathbf{y} \\ \mathbf{y}^\dagger & 1 \end{bmatrix} \succeq \mathbf{0}. \end{aligned}$$

Any feasible point for QCQP is also feasible for SDP, hence $B_{\text{SDR}}^{\text{Fro}} \triangleq \text{tr}(\mathbf{R}_0 \check{\mathbf{Y}}) + 2 \Re \{ \mathbf{q}_0^\dagger \check{\mathbf{y}} \} + \tau_0$.

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on MIMO aggregate channel gain

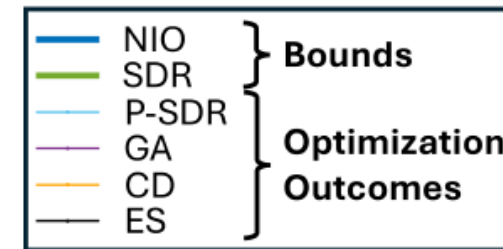
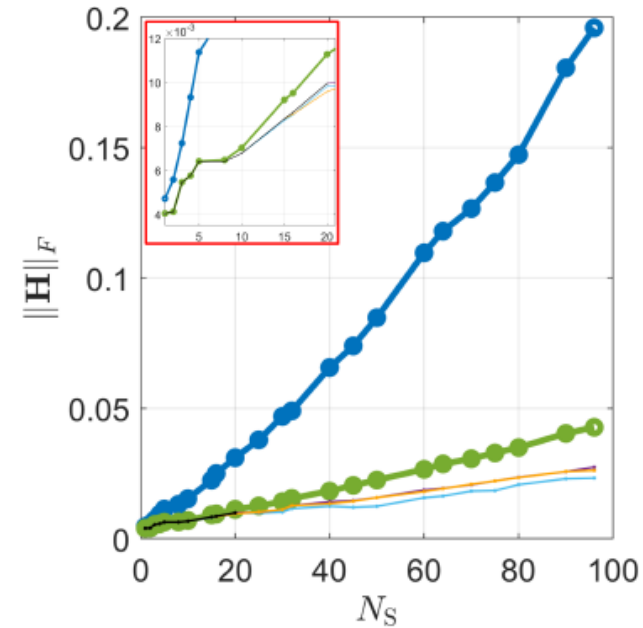
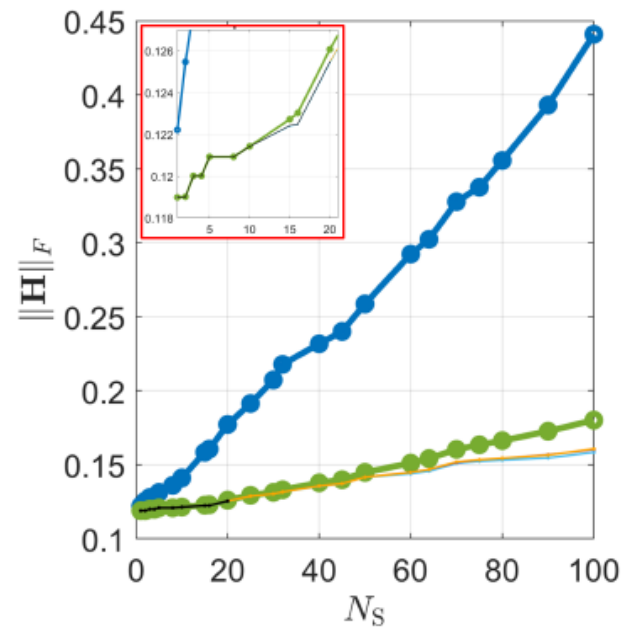
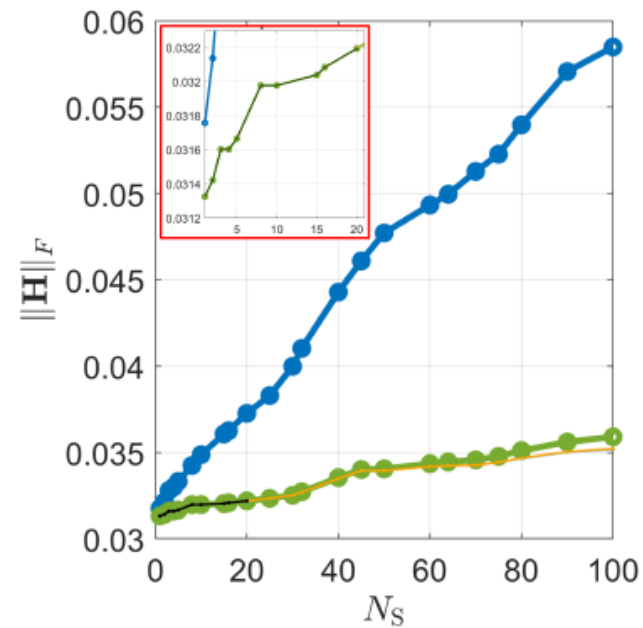
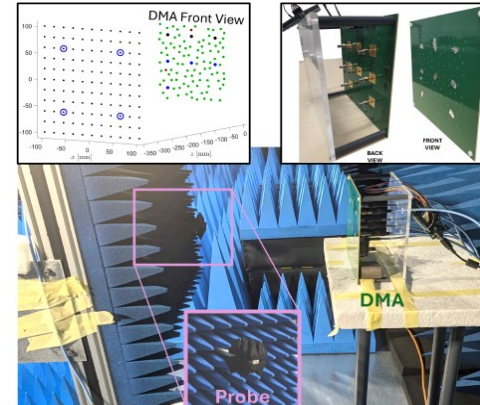
(a) RIS-Parametrized Free-Space MIMO Channel



(b) RIS-Parametrized Rich-Scattering MIMO Channel



(c) Multi-Feed-DMA Multi-User MIMO Channel



3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on MIMO operator synthesis

Goal: Maximize operator-synthesis fidelity $F(\mathbf{H}(\mathbf{r}), \mathbf{H}_{\text{des}}) \triangleq \frac{|\text{tr}(\mathbf{H}_{\text{des}}^\dagger \mathbf{H}(\mathbf{r}))|^2}{\|\mathbf{H}_{\text{des}}\|_F^2 \|\mathbf{H}(\mathbf{r})\|_F^2} \in [0, 1]$.

$$\begin{aligned} \mathbf{H}(\mathbf{r}) &= \mathbf{H}_0 + \mathbf{A} (\mathbf{I}_{N_S} - \Phi(\mathbf{r}) \Gamma)^{-1} \Phi(\mathbf{r}) \mathbf{B} \\ \Phi(\mathbf{r}) &= \text{diag}(\mathbf{r}) \\ \mathbf{r}(\mathbf{v}) &= \alpha \mathbf{1} + (\beta - \alpha) \mathbf{v} \end{aligned}$$

This objective is **fractional**-quadratic.

$$\begin{aligned} \max_{\mathbf{y}, \mathbf{Y}} \quad & \frac{\text{tr}(\mathbf{R}_1 \mathbf{Y}) + 2 \Re\{\mathbf{q}_1^\dagger \mathbf{y}\} + \tau_1}{h \text{tr}(\mathbf{R}_0 \mathbf{Y}) + 2h \Re\{\mathbf{q}_0^\dagger \mathbf{y}\} + h \tau_0} = \frac{f_n(\mathbf{y})}{f_d(\mathbf{y})} \\ \text{s.t.} \quad & \text{tr}(\mathbf{R}_{s,t} \mathbf{Y}) + \mathbf{y}^\dagger \mathbf{p}_{s,t} + \mathbf{q}_{s,t}^\dagger \mathbf{y} + \tau_{s,t} = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{1, \dots, N_T\}, \\ & \text{tr}(\mathbf{R}_{s,t}^{(i)} \mathbf{Y}) + \mathbf{y}^\dagger \mathbf{p}_{s,t}^{(i)} + (\mathbf{q}_{s,t}^{(i)})^\dagger \mathbf{y} + \tau_{s,t}^{(i)} = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{2, \dots, N_T\}, \\ & \quad \forall i \in \{1, 2\}, \\ & \mathbf{Y} = \mathbf{y} \mathbf{y}^\dagger. \end{aligned}$$

Introduce $\tilde{\mathbf{y}} = \sigma \mathbf{y}$ and $\tilde{\mathbf{Y}} = \sigma \mathbf{Y}$,
 where $\sigma = \frac{1}{f_d(\mathbf{y})}$
 (“Charnes–Cooper transformation”)

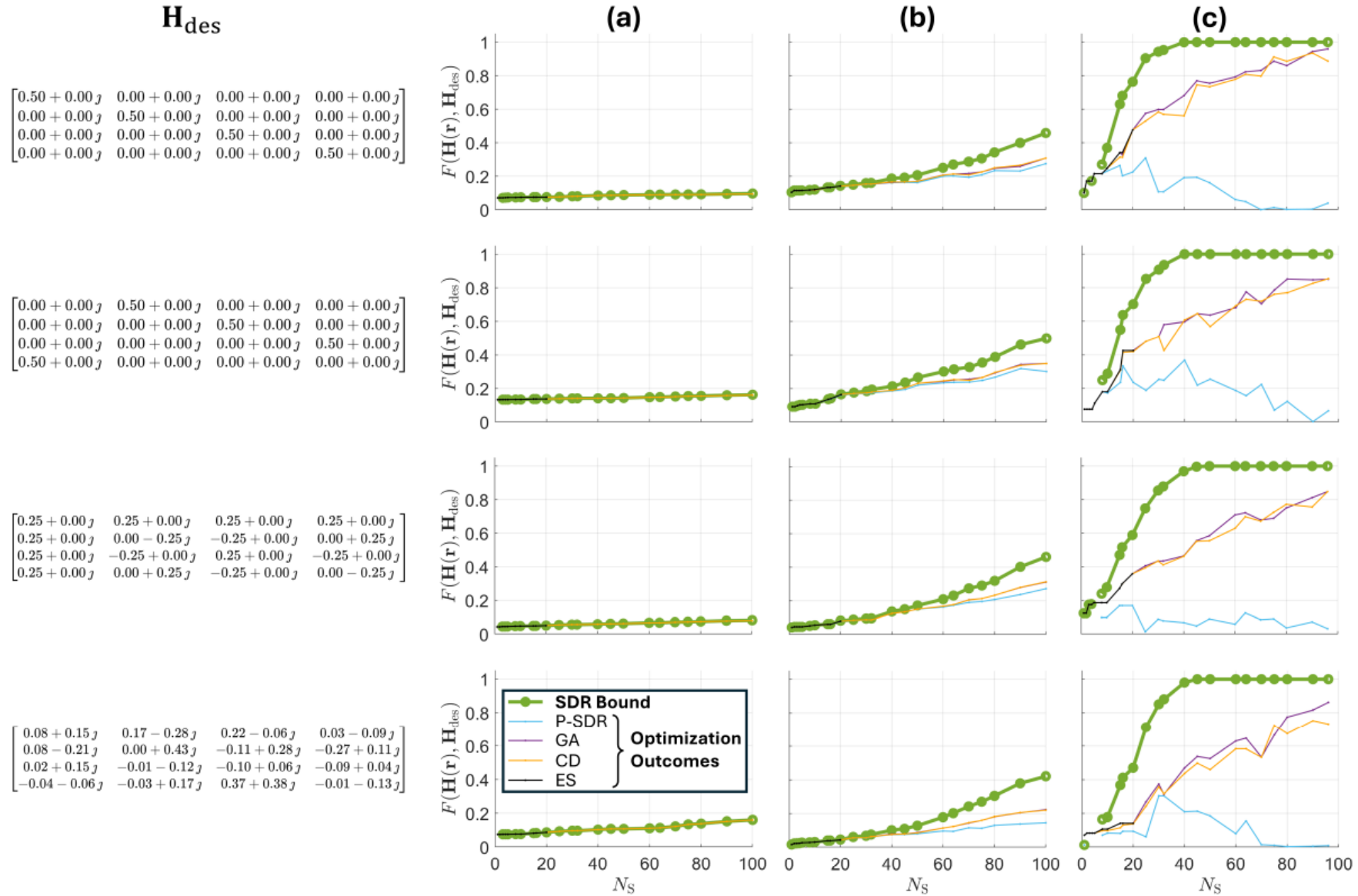
$$\begin{aligned} \max_{\tilde{\mathbf{y}}, \tilde{\mathbf{Y}}, \sigma} \quad & \text{tr}(\mathbf{R}_1 \tilde{\mathbf{Y}}) + 2 \Re\{\mathbf{q}_1^\dagger \tilde{\mathbf{y}}\} + \tau_1 \sigma \\ \text{s.t.} \quad & h \text{tr}(\mathbf{R}_0 \tilde{\mathbf{Y}}) + 2h \Re\{\mathbf{q}_0^\dagger \tilde{\mathbf{y}}\} + h \tau_0 \sigma = 1, \\ & \text{tr}(\mathbf{R}_{s,t} \tilde{\mathbf{Y}}) + \tilde{\mathbf{y}}^\dagger \mathbf{p}_{s,t} + \mathbf{q}_{s,t}^\dagger \tilde{\mathbf{y}} + \tau_{s,t} \sigma = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{1, \dots, N_T\}, \\ & \text{tr}(\mathbf{R}_{s,t}^{(i)} \tilde{\mathbf{Y}}) + \tilde{\mathbf{y}}^\dagger \mathbf{p}_{s,t}^{(i)} + (\mathbf{q}_{s,t}^{(i)})^\dagger \tilde{\mathbf{y}} + \tau_{s,t}^{(i)} \sigma = 0, \\ & \quad \forall s \in \{1, \dots, N_S\}, \\ & \quad \forall t \in \{2, \dots, N_T\}, \\ & \quad \forall i \in \{1, 2\}, \\ & \tilde{\mathbf{Y}} = \frac{1}{\sigma} \tilde{\mathbf{y}} \tilde{\mathbf{y}}^\dagger, \\ & \sigma > 0. \end{aligned}$$

Then, the usual SDR procedure yields an SDP, and ultimately

$$B_{\text{SDR}}^{\text{Fid}} \triangleq \text{tr}(\mathbf{R}_1 \tilde{\mathbf{Y}}) + 2 \Re\{\mathbf{q}_1^\dagger \tilde{\mathbf{y}}\} + \tau_1 \check{\sigma}.$$

3) Toward a prototype-aware EM information theory for reconfigurable microwave systems

Bounds on MIMO operator synthesis



Toward an Electromagnetic Information Theory for Reconfigurable Microwave Systems

Philipp del Hougne

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March 10, 2026 – RF Summit Finland

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Yale: A. Alhulaymi, O. D. Miller, A. D. Stone

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